A–1014

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MAT-509

M.Sc. Mathematics (MSCMT)

Mathematical Methods

Examination February, 2026

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *Two* (02) questions only.

1. Find the inverse Laplace transform of $\frac{s^3 + 6s^2 + 14s}{(s+2)^4}$.

2. Find the Fourier transform of $f(x) = \begin{cases} a - |x|, & |x| \le a \\ 0, & |x| > a \end{cases}$.

   Hence, show that :

$$\int_0^\infty \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}$$

3. Reduce the differential equation $\phi''(x) - 3\phi'(x) + 2\phi(x) = 4 \sin x$ with the conditions $\phi(0) = 1$, $\phi'(0) = -2$ into a non-homogeneous Volterra's integral equation of second kind. Conversely, derive the original differential equation with the initial conditions from the integral equation obtained.

4. Using Green's function, solve the boundary value problem $\frac{d^2u}{dx^2} - u = -2e^x$ with boundary conditions :

   $u(0) = u'(0)$, $u(l) + u'(l) = 0$.

5. Find the shortest distance between $y^2 = 4x$ and $(x - 9)^2 + y^2 = 4$.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Find the Laplace transform of $t^2e^t \sin 4t$.

2. Find the Fourier series of $f(x) = x^3$ in the interval $(-\pi, \pi)$.

3. Using Convolution theorem, find the inverse Laplace transform of $\frac{1}{(s+2)(s-1)}$.

4. Show that the function $\phi(x) = (1 + x^2)^{-3/2}$ is a solution of the Volterra integral equation :

$$\phi(x) = \frac{1}{1+x^2} - \int_0^x \frac{\xi}{1+x^2}\phi(\xi)d\xi$$

5. Convert the differential equation $y''(x) - 3y'(x) + 2y(x) = 4 \sin x$, with the initial condition $y(0) = 1$, $y'(0) = -2$ into Volterra's integral equation of second kind.

6. Find the Green's function for the boundary value problem $\frac{d^2u}{dx^2} - u(x) = 0$ with the conditions $u(0) = u(1) = 0$.

7. Find the extremals of the following functionals :

   (i) $\int_{x_0}^{x_1} (x + y')y'dx$

   (ii) $\int_{x_0}^{x_1} \left(\frac{y'^2}{x^3}\right)dx$

8. Find the shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$.

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