A–1012

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MAT-507

M.Sc. Mathematics (MSCMT)

Measure Theory

Examination February, 2026

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *Two* (02) questions only.

1. (a) Define countable and uncountable sets. Show that the set of all finite subsets of a countable set is countable.

(b) Define the concepts of ring and σ-ring. Prove that every σ-ring is a ring, but the converse need not hold.

2. (a) Define outer measure and measurable sets.

(b) State and prove that every closed interval $[a, b]$ is Lebesgue measurable and has measure $b - a$.

(c) Explain the construction of Lebesgue measure on $\mathbb{R}$.

3. (a) Define measurable functions and step functions. Prove that the sum and product of measurable functions are measurable.

(b) State and prove Fatou's Lemma.

(c) Explain the Dominated Convergence Theorem with an example.

4. (a) Define a function of bounded variation and prove that if f is of bounded variation, then f can be expressed as the difference of two increasing functions.

(b) Define convex functions and prove Jensen's inequality.

5. (a) Define L^p spaces. Prove Schwarz inequality and Hölder's inequality.

(b) State the Fubini's theorem and explain its application in double integrals.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *Four* (04) questions only.

1. Define σ-algebra and Boolean ring with examples.
2. State and prove that every subset of a null set is measurable.
3. Define simple functions. Show that every nonnegative measurable function can be approximated by an increasing sequence of simple functions.
4. Define convergence in measure and prove that convergence in L^p implies convergence in measure.
5. State and explain the Jordan decomposition theorem.
6. State the Lebesgue decomposition theorem and give its significance.
7. Differentiate between Riemann and Lebesgue integration with an example of a function that is Lebesgue integrable but not Riemann integrable.
8. State and prove the Weierstrass approximation theorem.

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