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MAT-506

M.Sc. Mathematics (MSCMT-23)

Topology

Examination February, 2026

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *Two* (02) questions only.

1. Prove that every compact subspace of a Hausdorff space is closed.

2. Prove that a closed and bounded subset (subspace) of $\mathbb{R}$ is compact.

3. Let $(T_\lambda : \lambda \in X\}$ where X is an arbitrary set, be a collection of topologies for X. Proof that the intersection $\cap \{T_\lambda : \lambda \in X\}$ is also a topology for X.

4. Show that for any family of topologies for X there exists a unique largest topology which is smaller than each member of the family.

5. Let $T_1 = \{\varnothing, \{1\}, X_1\}$ be a topology on $X_1 = \{1, 2, 3\}$ and

   $T_2 = \{\varnothing, X_2, \{a\}, \{b\} \{a, b\}, \{c, d\}, \{a, c, d\} \{b, c, d\}\}$

   be a topology for $X_2 = \{a, b, c, d\}$. Find a base for the product topology T?

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *Four* (04) questions only.

1. Define the following term :

   (i) Standard Topology

(ii) Lower Limit Topology

(iii) Upper Limit Topology

(iv) Subbasis for a Topology

2. Define the following term in topological space :

(i) Closed Set

(ii) Interior of a Set

(iii) Closure of a Set

(iv) Derived Set

3. In any topological space, prove that $A \cup D(A)$ is closed.

4. (i) Let $X = \{a, b, c\}$. How many different topologies on X. Explain them.

(ii) Check that whether $T_1 = \{X, \varnothing, \{a\}\}$ and $T_2 = \{X, \varnothing, \{b\}\}$ is a topology for $X = \{a, b, c\}$.

5. Let $X = \{a, b, c\}$ and $T = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\}$. Then prove that (X, T) is not a Hausdorff space.

6. State The Urysohn Lemma and Tietze Extension Theorem.

7. Describe T_1 – Spaces and Hausdorff Topological Spaces with examples.

8. Explain Limit Point Compactness and Sequentially Compact with examples.

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