A–1010

Total Pages : 3 **Roll No. ....**

MAT-505

M.Sc. Math (MSCMT)

Advanced Linear Algebra

Examination February, 2026

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *Two* (02) questions only.

1. Defined the following: vector space over any field, Spanning set, linear dependence and linear independence. Also, prove that that intersection of two subspace of a vector space is a subspace.

2. If W_1 and W_2 are finite dimensional subspace of a vector space V, then show that $W_1 + W_2$ is finite dimensional and $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$.

3. Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$, $\alpha_3 = (0, -3, 2)$ form a basis for R^3. Express each of the standard basis vectors as a linear combination of α_1, α_2, α_3.

4. Prove that every linear operator T on a finite dimensional complex inner product space V can be uniquely expressed as $T = T_1 + iT_2$, where T_1 & T_2 are self-adjoint linear operators on V.

5. State and prove the Cauchy Schwartz inequality.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *Four* (04) questions only.

1. Show that the vectors (1, 2, 1), (2, 1, 0) and (1, –1, 2) form a basis of R^3.

2. Show that the mapping $T : V_2(R) \to V_3(R)$ defined as, $T(a, b) = (a + b, a - b, b)$ is a linear transformation form $V_3(R)$ to.

3. If T is a linear transformation from U(F) to V(F) then prove that kernel of T 'or' null space of T is a subspace of U(F).

4. Find the rank and nullity of the matrix :

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$$

5. Find the coordinate of the vector (2, 1, –6) of R^3 relative of the basis $\alpha_1 = (1, 1, 2)$, $\alpha_2 = (3, -1, 0)$ $\alpha_3 = (2, 0, -1)$

6. Show that the necessary and sufficient condition for a vector space V(F) to be a direct sum of its two subspace W_1 and W_2 are that:

   (a) $V = W_1 + W_2$

   (b) $W_1 \cap W_2 = \{0\}$ *i.e.*, W_1 and W_2 are disjoint.

7. Let T be the lincar operator on R^3 which is represented in the standard ordered basis by the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$ · Find the minimal polynomial for T.

8. Let T be a linear operator on a finite-dimensional vector space V(F) Then $\alpha \in F$ is an eigenvalue of T if and only if $T - \alpha I$ is singular.

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