A–1007

Total Pages : 3 **Roll No. ....**

MAT-502

Mathematics (MSCMAT/MAMT)

Advanced Real Analysis

Examination February, 2026

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *Two* (02) questions only.

1. State and prove Rolle's theorem with example.

2. Define Raabe's test and test the convergence of the following series :

(i) $\frac{2}{7}+\frac{2.5}{7.10}+\frac{2.5.8}{7.10.13}+.........$

(ii) $\frac{1}{2}+\frac{1.3}{2.5}+\frac{1.3.5.}{2.5.8}+..........$

3. Show that $f(x) = 2x + 1$ is integrable on [1, 2] and $\int_1^2 (2x+1)dx = 4$.

4. Define uniform convergence and show that the series $\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}+\frac{8x^7}{1+x^8}+.....$ is uniformly convergent in (–1, 1).

5. State and prove Banach Contraction Theorem.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *Four* (04) questions only.

1. Define D'Alembert's ratio test and test the convergence of the series :

$$\left(\frac{1}{3}\right)^2+\left(\frac{1.2}{3.5}\right)^2+\left(\frac{1.2.3.}{3.5.7}\right)^2+.....$$

2. Prove that the set N × N is countable.

3. Show thatthe function $f(x)$, where :

$$f(x)=\begin{cases} x^2-1, & x\geq 1 \\ 1-x, & x<1 \end{cases}$$

has no derivative at $x = 1$.

4. Compute $L(P, f)$ and $U(P, f)$ for the function f defined by $f(x) = x^2$ on $[0, 1]$ and $P = \left\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\right\}$.

5. Examine the convergence of the integral $\int_0^1 \log x \, dx$.

6. Show that the sequence $\{f_n\}$, where $f_n(x) = \tan^{-1} nx$, $x \geq 0$ is uniformly convergent in any interval $[a, b]$ $a > 0$ but is only pointwise convergent in $[0, b]$.

7. Prove that every closed sphere is a closed set in a metric space (X, d).

8. Define the following:

   (i) Metric spaces

   (ii) Non-expansive mappings

   (iii) Limit point of a sequences

   (iv) Measurable sets

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