

**A-1006**

Total Pages : 3

Roll No. ....

**MAT-501**

**Mathematics (MSCMT-23)**

**Advanced Abstract Algebra**

Examination February, 2026

Time : 2:00 Hrs.

Max. Marks : 70

**Note** :- This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. *Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.*

**Section-A**

**Long Answer Type Questions**       $2 \times 19 = 38$

**Note** :- Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *Two* (02) questions only.

1. If  $G$  be a group and  $H$  is the subgroup of  $G$ . Then the following statement are equivalent.
  - (i) The subgroup  $H$  is normal in  $G$

**A-1006**

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- (ii) For all  $a \in G$ ,  $aHa^{-1} \subseteq H$
  - (iii) For all  $a \in G$ ,  $aHa^{-1} = H$
2. Using Cayley's theorem find the permutation group which is isomorphic to the  $D_4$ .
  3. The necessary and sufficient conditions for a non-empty subset  $S$  of a ring  $R$  to be a subring of  $R$  are :
    - (i)  $a, b \in S \Rightarrow a-b \in S$
    - (ii)  $a, b \in S \Rightarrow ab \in S$
  4. Prove that commutative ring without zero divisors can be imbedded in a field.
  5. Show that A factorization domain is a unique factorization domain if and only if every irreducible element is prime.

### Section–B

#### Short Answer Type Questions 4×8=32

**Note** :- Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *Four* (04) questions only.

1. Find the class equation for the group  $S_3$ .
2. State and prove Sylow's First Theorem.
3. Define Normal and Composition series with Example.

4. Show that Every nilpotent group is solvable.
5. Prove that intersection of two ideal of a ring is also an ideal of ring.
6. Prove that Every field is an integral domain.
7. Define the following terms with examples :
  - (a) Group
  - (b) Ring
  - (c) Integral domain
  - (d) Field
8. Prove that every ideal in  $F[x]$  is principal ideal, where  $F$  is principal ideal.

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