A–1053

Total Pages : 4 **Roll No. ....**

MAMT-09

M.A./M.Sc. Mathematics (MAMT/MSCMT)

Integral Transforms and Integral Equations

Examination, 2026 (Feb.)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Write the statement of convolution theorem. Let $f(t)$ and $g(t)$ be two functions of class A and let $L^{-1}\left[\bar{f}(p); t\right] = f(t)$ and $L^{-1}\left[\bar{g}(p); t\right] = g(t)$. Then prove that :

$$L^{-1}\left[\bar{f}(p).\bar{g}(p); t\right] = \int_0^t f(u)g(t-u)du = f * g$$

2. Find the solution of the partial differential equation of :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

where $0 < x < 1$, $t > 0$ together with the conditions $u(x, 0) = 3 \sin 2\pi x$, $u(0, t) = 0$, $u(1, t) = 0$.

3. Find $f(t)$ if its Fourier sine transform is $\dfrac{p}{(1+p^2)}$.

4. Find the eigenvalues and eigenfunction of the homogeneous integral equation :

$$g(x) = \lambda \int_0^{\pi} \left[\cos^2 x \cos 2t + \cos 3x \cos^3 t\right] g(t)dt$$

5. Show that the integral equation :

$$g(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} \sin(x+t)g(t)dt$$

possesses no solution for $f(x) = x$, but that is possesses infinitely many solutions when $f(x) = 1$.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Find the Laplace transform of the following function :

   (i) $e^{-2t} - \sin 5t + 4 \cos 7t + 9t^3 - 5$

   (ii) $t^2 e^{3t}$

2. If $L^{-1}[\bar{f}(p); t] = f(t),$ then prove that :

$$L^{-1}[\bar{f}(ap); t] = \frac{1}{a} f\left(\frac{t}{a}\right), \; a > 0$$

3. Evaluate :

$$L^{-1}\left[e^{-a\sqrt{p}}\right]$$

4. Solve the differential equation by using the Laplace/ inverse Laplace transform :

$$(D^2 + 9)y = \cos 2t \text{ if } y(0) = 1, \; y\left(\frac{\pi}{2}\right) = -1$$

5. State and prove the relation between Fourier transform and Laplace transform.

6. Prove that if n is a positive integer :

$$\mathrm{M}\left[\left(x\frac{d}{dx}\right)^n f(x);\ p\right] = (-1)^n p^n \mathrm{F}(p),$$

where $\mathrm{M}\{f(x); p\} = \mathrm{F}(p)$.

7. Find the Hankel transform of the following functions :

(i) $\dfrac{\cos ax}{x}$

(ii) $\dfrac{\sin ax}{x}$

Where, $x\mathrm{J}_0(px)$ as the kernel.

8. Show that the function $g(x) = xe^x$ is a solution of the Volterra integral equation :

$$g(x) = \sin x + 2\int_0^x \cos(x-t)g(t)dt$$

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