A–1051

Total Pages : 3 **Roll No. .....**

MAMT-06

M.A./M.Sc. Mathematics (MAMT/MSCMT)

Analysis and Advanced Calculus

Examination, 2026 (Feb.)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. State and proved Holder's inequality.

2. If T be an linear transformation from a Normed linear Space N into the Normed linear show that Space N′ then show that the following statement are equivalent :

   (i) T is continuous.

   (ii) T is continuous at the origin *i.e.*, $x_n \to 0 \Rightarrow$, $T(x_n) \to 0$.

   (iii) T is bounded *i.e.*, their exist real $k \geq 0$ s.t $\| T(x) \| \leq k \| x \|$ for all $x \in N$.

   (iv) If $S = \{x : \| x \| \leq 1\}$ is the closed unit sphere in N, then its image T(s) is bounded set in N′.

3. State and prove Closed Graph theorem.

4. If M be a linear subspace of a Normed linear Space N and f is a functional defined on M, show that then show that f can be extended to a functional fo defined on the whole space N s.t. $\| f_0 \| = \| f \|$.

5. If B is a complex Banach space whose norm obeys the parallelogram law, and if an inner product is defined on B by :

$$4(x, y) = \| x + y \|^2 + \| x - y \|^2 + i\| x + iy \|^2 - i\| x - iy \|^2$$

then prove that B is a Hilbert space.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. If N is a Normed linear space and $x, y \in N$, then prove that :

$$| \| x \| - \| y \| | \leq \| x - y \|$$

2. Let N and N′ be Normed Linear Space over the same scaler field and let T be a linear transformation of N into N′. Then show that T is bounded if it is continuous.

3. Prove that the weak limit of a sequence is unique.

4. Prove that every compact subset of a normed space is bounded but the converse is not true.

5. An orthonormal set S in a Hilbert space H is complete iff $x \downarrow s \Rightarrow x = 0 \ \forall \ x \in H$.

6. An operator Ton H is self-adjoint, then show that $(Tx, y) = (x, Ty) \ \forall \ x \in H$ and conversely.

7. If T_1 and T_2 are Normal Operators on H with the property that either commutes with prou adjoint of the other, then T_1 and T_2 are normal.

8. If x is an eigenvector of T, then prove that x cannot correspond more than one eigenvalue of T.

****