A-0644

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MT–609

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Integral Equations)

4th Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Reduce $y'' + \lambda xy = f(x)$, $y(0) = 1$, $y'(0) = 0$ into an integral equation.

2. Solve :

$$y(x) = 1 + \int_0^1 (1 + e^{x+t}) y(t) dt$$

3. Show that eigen functions of a symmetric kernel for distinct eigen values are orthogonal.

4. Solve :

$$y(x) = \frac{5x}{6} + \frac{1}{2} \int_0^1 xty(t) dt$$

5. Solve :

$$y(x) = f(x) + \int_0^x \frac{2 + \cos x}{2 + \cos t} y(t) dt$$

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Find λ if $u(x) = e^x(2x - \lambda)$ is a solution of :

$$u(x) + 2 \int_0^1 e^{x-t} u(t) dt = 2xe^x$$

2. Discuss Boundary Value Problem with example.

3. Show that :

$$u(x) - \lambda \int_0^1 t(3x-2)u(t)dt = 0$$

has no eigen function.

4. If $K(x, t) = e^{x-t}$. Show that :

$$R(x, t, \lambda) = e^{\lambda(x-t)} K(x, t)$$

5. Solve :

$$u(x) = 1 + \int_0^x u(t)dt$$

6. Discuss various kinds of Volterra integral equations.

7. If $K(x, t) = \sin x \cos t$. Show that :

$$R(x, t; \lambda) = K(x, t)$$

8. Show that :

$$B_p(x, t) = 0 \text{ for all } p \text{ if } K(x, t) = 2x - t.$$

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