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MT–608

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Numerical Analysis-II)

4th Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Use Runge-Kutta method of forth order to find the value of y, when $h = 0.5$ for the equation :

$$\frac{dy}{dt} = te^{3t} - 2y,\ 0 \le t \le 1 : y(0) = 0$$

2. Use the Adams-Bashforth methods to approximate the solutions to the following initial-value problems. In each case use exact starting values, and compare the results to the actual values :

$$\frac{dy}{dt} = 1 + \frac{y}{t},\ 1 \le t \le 2 : y(1) = 2, \text{ with } h = 0.2$$

3. Use Taylor's method of order two to approximate the solutions for each of the following initial-value problems :

(i) $\dfrac{dy}{dt} = e^{t-y},\ 0 \le t \le 1 : y(0) = 1$, with $h = 0.5$

(ii) $\dfrac{dy}{dt} = \sin t + e^{-t},\ 0 \le t \le 1 : y(0) = 0$, with $h = 0.5$

4. Given the initial-value problem :

$$\frac{dy}{dt} = \frac{2}{t}y + t^2 e^t,\ 1 \le t \le 2 : y(1) = 0$$

with exact solution $y(t) = t^2(e^t - e)$:

Use Taylor's method of order two with $h = 0.1$ to approximate the solution, and compare it with the actual values of y.

5. Use Picard's method to find two successive approximate solutions of the initial value problem :

$$\frac{dy}{dx} = \frac{y-x}{y+x}; y(0) = 1$$

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Compute values of $y(1.1)$ and $y(1.2)$ on solving the following initial value problem, using Runge-Kutta methods of order 4 :

$$y'' + \frac{y'}{x} + y = 0, \text{ with } y(1) = 0.77 \text{ and } y'(1) = -0.44$$

2. Given :

$$\frac{dy}{dx} = \frac{(1+x^2)y^2}{2}$$

and $y(0) = 1$, $y(0.1) = 1.06$, $y(0.2) = 1.12$, $y(0.3) = 1.21$.

Compute $y(0.4)$ by Milne's predictor-corrector method.

3. Fit a second-degree polynomial of y on x to the following data :

x	***y***
50	12
70	15
100	21
120	25

4. Fit a relation of the form R = $a + b$V + cV^2 to the following data, where V is the velocity in km/hr and R is the resistance in km/quintal. Estimate R, when V = 90 :

V	**R**
20	5.5
40	9.1
60	14.9
80	22.8
100	33.3
120	46.0

5. Use Runge-Kutta method of forth order to find the value of y, when x = 0.2 for the equation :

$$\frac{dy}{dx} = \frac{y-x}{y+x} . y(0) = 1, \text{ Take } h = 0.2$$

6. Form the Taylor series solution of the initial value problem :

$$\frac{dy}{dx} = xy + 1; \ y(0) = 1$$

upto five terms and hence compute $y(0.1)$ and $y(0.2)$, correct to four decimal places.

7. Given :

$$x\frac{dy}{dx} = x - y^2, \ y(2) = 1$$

evaluate $y(2.1)$, $y(2.2)$ and $y(2.3)$ correct to four decimal places using Taylor series method.

8. Explain stability analysis of :

(a) Euler's Method

(b) Runge-Kutta method of order two

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