A-0641

Total Pages : 3 **Roll No. ....**

MT–606

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Analysis and Advanced Calculus-II)

4th Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Prove that if T is a normal operator on a Hilbert space H, then x is an eigenvector of T with eigenvalue λ, iff x is an eigenvector of T* with λ as eigenvalue.

2. Prove that If T is a normal operator on Hilbert space H then each eigenspace of T reduces T.

3. State and prove Inverse function Theorem.

4. Prove that if f be a regulated function on a compact interval $[a, b]$ of R into R such that $a < b$ and for all t in $[a, b]$, $f(t) \geq 0$. Then :

$$\int_a^b f(t)dt \geq 0$$

5. Prove that every scalar multiple of self- adjoint operator is also normal.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Prove that a closed linear subspace M of a Hilbert space H reduces an operator T if and only if M is invariant under both T and T*.

2. Prove that If T is an arbitrary operator on Hilbert space H, then :

$$T = 0 \text{ iff } (Tx, y) = 0; x \in H \text{ iff } T = 0$$

3. Define Lipschitz's Property and Existence Theorem for differential equation.

4. State and proof Taylor's formula with Lagrange's form of Reminder.

5. Define eigen values, eigen vectors and Projection on a Hilbert Space.

6. Prove that if X and Y be two Banach spaces over the same field K. In the set of all functions tangential to a function f at $v \in V$, there is at most one function $\phi : X \to Y$, of the form $\phi(x) = f(v) + g(x - v)$, where $g : X \to Y$, is linear, where V is a non-empty open subset of X.

7. Prove that if T is normal operator on a Hilbert space H, then eigenspaces of T are pairwise orthogonal.

8. State and Prove Global Uniqueness Theorem.

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