A-0636

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MT–601

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Analysis and Advanced Calculus-I)

3rd Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. State and prove Minkowski inequality.

2. Prove that in a finite dimensional’space, the notions of weak and strong convergence are equivalent.

3. If N be a non-zero normed space and let :

$$S = \{x \in N : \| x \| \leq 1\}$$

be a linear subspace of N. Then prove that N is a Banach space iff S is complete.

4. State and prove Hahn-Banach Theorem.

5. If H be a Hilbert space and (e_i) be an orthonormal set in H. Then show that following are equivalent :

(i) $\{e_i\}$ is complete

(ii) $x \perp \{e_i\} \Rightarrow x = 0.$

(iii) If x is an arbitrary vector in H, then :

$$x = \sum \langle x, e_i \rangle e_i \, .$$

(iv) If x is an arbitrary vector in H, then :

$$\| x \|^2 = \sum | \langle x, e_i \rangle |^2$$

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Show that the linear spaces R^n(Euclidean) of n-tuples $x = (x_1, x_2, ..., x_n)$ of real numbers are Banach Space under the norm :

$$\| x \| = \left\{ \sum_{i=1}^{n} | x_i |^2 \right\}^{1/2}$$

2. Show that every complete subspace of a normed linear space in closed.

3. If T be a linear transformation of normed space N into normed space N′, then inverse of T i.e., T^{-1} exists and is continuous on its domain of definition *iff* $\exists$ *a* constant $K \geq 0$, s.t. $\| Tx \| \geq K \| x \|, \forall x \in N$.

4. State and prove open mapping theorem.

5. Prove that every Hilbert space is an inner product space but converse not necessarily true.

6. Define conjugate of an operator in a normed space and show that the adjoint of a continuous linear map T is linear.

7. A If x and y are any two vectors in a Hilbert space H then show that :

   (i) $\| x + y\|^2 - \| x - y \|^2 = 4Re\langle x, y\rangle$

   (ii) $(x, y) = \text{Re}(x, y) + i\text{R}e\langle x, iy\rangle$

8. Prove that Hilbert space (l_2, d) is complete.

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