A-0635

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MT–510

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Mechanics-II)

2nd Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. If initially the axis of the top is horizontal and it is set spinning with angular velocity w in a horizontal plane, prove that the axis will start to rise if $nCw > mgh$ and that, when $nCw = 2mgh$, the axis will rise to an angular distance $\cos^{-1}\left(\frac{Aw}{nc}\right)$, provided that $Aw < nC$, and will there be at instantaneous rest. A, C and n have their usual meanings.

2. A particle moves in a straight line with central acceleration μx between two points x_0 and x_1 in the prescribed time $t_1 - t_0$. Show that Hamilton's principle function S is :

$$\frac{\sqrt{\mu}\{(x_1^2 + x_0^2)\cos(t_1 - t_0)\sqrt{\mu} - 2x_1x_0\}}{2\sin(t_1 - t_0)\sqrt{\mu}}$$

3. (a) Find the equation of the stream lines for the flow :

$$\vec{q} = x\hat{i} - y\hat{j}$$

   (b) Find the equation of the stream lines passing through the point (1, 1, 1) for an incompressible flow :

$$\vec{q} = 2x\hat{i} - y\hat{j} - z\hat{k}$$

4. Derive equation of Continuity by the Lagrangian Method.

5. Derive equation of continuity in cartesian coordinates system.

Section–B

Short Answer Type Questions 4×8=32

Note :– Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Explain the following :

   (a) Ideal fluids

   (b) Stream lines

   (c) Path lines

2. State and prove Bernoulli's theorem.

3. A pulse travelling along a fine straight uniform tube filled with gas causes the density at time t and distance x from the origin where the velocity is u_0 to become $\rho_0\phi(vt - x)$. Prove that velocity u is given by

$$v + \frac{(u_0 - v)\phi(vt)}{\phi(vt - x)}$$

4. State and prove Equations of motion under impulsive force in Cartesian form.

5. (a) Define axis of the doublet.

   (b) Establish Cauchy Riemann equation in Cartesian coordinates.

6. A source S and a sink T of equal strength m are situated within the space bounded by a circle whose centre is O. If S and T are at equal distances from O an opposite side of it and on the same diameter AOA′. Show that the velocity of the liquid at any point P is :

$$2m.\frac{OS^2+OA^2}{OS}.\frac{PA.PA'}{PS.PS'.PT.PT'}$$

where S′ and T′ are the inverse points of S and T with respect to the circle.

7. Establish an equation for steady motion of a top.

8. A heavy bead of mass m is freely movable on a smooth circular wire of radius a, which is made to rotate about the vertical diameter with spin w, θ being the angle made by the radius through the bead at any time with the downwards vertical, prove that the action A is :

$$A=\int_{\theta_1}^{\theta_2} ma^2\left\{\frac{2H}{ma^2}+\frac{2g}{a}\cos\theta+w^2\sin^2\theta\right\}^{1/2}d\theta$$

where H is the Hamiltonian of the system.

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