A-0634

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MT–509

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Differential Geometry and Tensor-II)

2nd Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Derive the Canonical equations of a geodesic on the surface :

$$r = r(u, v)$$

2. State and Prove Gauss's Formulae.

3. Define :

   (a) Covariant vectors

   (b) Invariant

   (c) Contravariant tensor of rank two

   (d) Zero tensor

   (e) Symmetric tensor

4. Prove that :

   (a) $[ij, m] = g_{lm} \begin{bmatrix} l \\ ij \end{bmatrix}$

   (b) $\dfrac{\partial g_{ik}}{\partial x^j} = [ij, k] + [kj, i]$

   (c) $\dfrac{\partial g^{mk}}{\partial x^l} = -g^{mi} \begin{Bmatrix} k \\ il \end{Bmatrix} - g^{ki} \begin{Bmatrix} m \\ il \end{Bmatrix}$

5. (a) Define :

   (i) Isotropic point

   (ii) Riemann-Christoffel tensor.

(b) The necessary and sufficient condition for a space V_N to be flat is that the Riemann-Christoffel tensor be identically zero, i.e., $R^{\alpha}_{ijk} = 0$.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. State and Prove Clairant's theorem.

2. (a) Define parallel surfaces

   (b) State Bonnets theorem on parallel surfaces.

3. Prove that the law of transformation of a contravariant vector is transitive.

4. If a metric of aV_3 is given by :

$$ds^2 = 5\,(dx^1)^2 + 3(dx^2)^2 + 4\,(dx^3)^2 - 6(dx^1)(dx^2) + 4(dx^2)(dx^3)$$

   find (i) g and (ii) g^{ij}.

5. State and prove Ricci's theorem.

6. Prove that if two unit vectors A^i and B^i are defined along a curve C such that their intrinsic derivatives along C are zero, show that the angle between them is constant.

7. Show that on the surface of a sphere, all great circles are geodesies while no other circle is a geodesic.

8. State and prove fundamental theorem of Riemannian geometry.

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