A-0633

Total Pages : 4 **Roll No. ....**

MT–508

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Special Functions)

2nd Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note **:–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

Section–A

Long Answer Type Questions $2 \times 19 = 38$

Note **:–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Solve :

$$5x^2 . \frac{d^2y}{dx^2} + x.(1+x)\frac{dy}{dx} - y = 0$$

by Frobenius method.

2. (a) Find the expression for Legendre's differential equation.

(b) Show that :

$$\int_{-1}^{1} [P_n(x)]^2\, dx = \frac{2}{2n+1}, \quad \text{if } m = n$$

3. Show that :

(a) $\sin z = 2J_1 - 2J_3 + 2J_5 - 2J_7 + ...$

(b) $\cos z = J_0 - 2J_2 + 2J_4 - 2J_6 + ...$

4. Show that :

$$H_n(x) = (-1)^n e^{x^2} . \frac{d^n e^{-x^2}}{dx^n}$$

5. Prove that :

(a) $L_n^n(0) = (-1)^n$

(b) $L_n(0) = 1$

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Write a short note on the followings :

   (a) Regular and singular points

   (b) Contiguous function

2. Show that :

$$ {}_1F_1(a; b; z) = e^z . {}_1F_1(b - a; b; -z) $$

3. Prove that :

$$ (2n + 1)x . P_n = (n + 1) P_{n+1} + n . P_{n-1} $$

4. Derive the differential equation for the Gauss Hypergeometric function.

5. Show that the linear transformation formula for the Gauss hypergeometric function is :

$$ {}_2F_1(a, b; c; z) = (1 - z)^{-a} {}_2F_1(a, c - b; c; z/(z - 1)) $$

6. Show that :

$$J_{-n}(z) = (-1)^n J_n(z)$$

where n is a positive integer.

7. Prove :

$$2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$$

8. Show that :

$$(n + 1)L_{n+1}(x) = (2n + 1 - x)L_n(x) - nL_{n-1}(x)$$

****