A-0632

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MT–507

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Topology)

2nd Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Define a **topological space**. Let X be a set and T_1 and T_2 be two topologies in X. When do we say T_1 is **weaker** than T_2 ? Let X = $\mathbb{R}$ and define two topologies in X :

   - The **usual topology** T_U on X as the topology obtained by taking as a base the collection of all open intervals (*a*, *b*).

   - The K-**topology** T_K on X as the topology obtained by taking as a base the collection of all open intervals (*a*, *b*) together with all sets of the form $(a, b)\backslash K$, where $K = \{1/n : n \in N\}$.

   Which topology between T_K and T_U is weaker ? justify your answer in full details.

2. What are **subspaces** in topology ? Prove that the subspace topology on a subset of a topological space is the finest topology on that subset that makes the inclusion map continuous.

3. Explain the separation axioms T_0, T_1, T_2, T_3 and T_4. Provide examples of spaces that satisfy each of these axioms.

4. Discuss compact spaces in topology. Prove that every compact Hausdorff space is T_4.

5. Define continuous mappings between topological spaces. Prove that the preimage of an open set under a continuous map is open.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. What is a locally compact space ? Give an example

2. Define Tychnoff's one-point compactification. What is the one point compactification of $\mathbb{R}$?

3. Define product space and give an example. Show that the product of two compact spaces is compact.

4. What is a quotient space ? Provide an example and discuss the properties of quotient spaces.

5. Define nets in topology. Prove or disprove : every convergent sequence is a net, but not every net is a sequence.

6. Explain the concept of filters in topology. Give an example of a filter and describe its properties.

7. What is meant by a connected space ? Prove that the union of two connected spaces is connected if they have a non-empty intersection.

8. Show that the continuous image of a connected set is connected.

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