**A-0631**

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# MT–506

## M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

## (Advanced Algebra-II)

2nd Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

***Note*** **:–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

## Section–A

**Long Answer Type Questions** $2 \times 19 = 38$

***Note*** **:–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. If K is a finite extension of a field F, then the group G(K|F) of F-automorphism of K is finite, and :

$$o[G(K|F)] = \{K : F\}$$

2. Let K be a Galois extension of a field F. Then the set of all F-automorphism of K forms a group with respect to the operation of functions composition.

3. Define the Matrix of a linear map. Let $t : R^3 \to R^3$ be a linear transformation such that :

$$t(a, b, c) = (3a + c, -2a + b, -a + 2b + 4c)$$

What is the matrix of t in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3\}$ where ?

$$\alpha_1 = (1, 0, 1), \alpha_2 = (-1, 2, 1), \alpha_3 = (2, 1, 1)$$

4. Define :

   (a) Eigen values

   (b) Eigen vector of linear transformation

   (c) Similar matrices

   (d) Column rank of a matrix

5. (a) Define a real inner product space with an example.

   (b) State and prove the Cauchy- Schwartz inequality

## Section–B

**Short Answer Type Questions** 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1 . Define :

(a) Splitting fields

(b) Normal Extension

2. Prove that Similar matrices have the same characteristic polynomial and hence the same eigen values.

3. Let A and B be two similar matrices over the same field F. Prove that det(A) = det(B).

4. If W is a subspace of an inner product space $R^3$ spanned by $B_1 = \{(1, 0, 1), (1, 2, -2)\}$, then find a basis of orthogonal complement $W^{\perp}$.

5. Prove that if $t_1$ and $t_2$ are linear transformation of finite dimensional inner product space V to V′, then :

$$(t_1 + t_2)^* = t_1^* + t_2^*$$

6. Let V be a finite dimensional inner product space. Prove that a linear transformation $t : V \to V$ is orthogonal if and only if its matrix relative to an orthonormal basis is orthogonal.

7. Prove that the determinant of an orthogonal matrix is $\pm 1$.

8. Define :

   (a) Field of rational functions

   (b) Galois extension

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