A-0628

Total Pages : 3 **Roll No. ....**

MT–503

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Differential Equation and Calculus of Variation)

1st Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions $2 \times 19 = 38$

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Discuss the importance of Riccati's equation.

2. Solve $y' = 2 - 2y + y^2$ if one particular solution is $1 + \tan x$.

3. Solve :

$$z(y + z)dx + z(t - x)dy + y(x - t)dz + y(y + z)dt = 0$$

4. A tightly stretched sting which has fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = k\sin^3\left(\frac{\pi x}{l}\right)$. It is released from rest from this position. Find the displacement $y\,(x,\, t)$.

5. Find the curve with fixed boundary revolves such that its rotation about x-axis generate minimal surface area.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section ‘B’ contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Solve :

$$y^3 \frac{d^2y}{dx^2} = c$$

2. Solve :

$$(yz + xyz)dx + (zx + xyz)dy + (xy + xyz\}dz = 0$$

3. Solve :

$$(y^2 + z^2 - x^2)dx - 2xydy - 2xzdz = 0$$

4. Find the surface passing through the circle $z = 0$, $x^2 + y^2 = 1$ and satisfying the differential equation $s = 8xy$.

5. Solve $t - qx = x^2$.

6. Determine the nature of $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$.

7. Discuss geometrical meaning of $Pdx + Qdy + Rdz = 0$.

8. Find the extremal of the functional :

$$I = \int_0^1 (1 + y''^2)\, dx$$

under the conditions $y(0) = 0$, $y(1) = 1 = y'(0) = y'(1)$.