A-0627

Total Pages : 3 **Roll No. ....**

MT–502

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Real Analysis)

1st Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Prove that the outer measure of an interval is it's length.

2. Prove that a continuous function defined on a measurable set is always measurable. However its converse is not always true.

3. Proof that the set R of real numbers is uncountable.

4. Proof that L_2 is a normed linear space.

5. Proof that if $\{f_n\} \to f$ in measure on E, then there is a subsequence $\{f_{nk}\}$ that converges pointwise a.e. on E to f.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Define the following :

   (i) Hilbert space and example.

   (ii) Orthogonal elements

   (iii) Orthonormal system

   (iv) State Riesz-Fisher theorem

2. Give **yes/no** that the following set are countable :

(i) The set $\mathbb{Q}$ of a rational number.

(ii) The set [0, 1].

(iii) The set of all irrational number.

(iv) The sets of natural numbers $\mathbb{N}$ and integers $\mathbb{Z}$

3. Prove that there are disjoint sets of real numbers A and B for which :

$$m^*(A \cup B) < m^*(A) + m^*(B)$$

4. Prove that every closed set is measurable.

5. Show that L^p – space is a metric space.

6. Let $X = \{a, b, c, d\}$ and let $B = \{\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}\}$ then show that B forms a Boolean algebra.

7. Show that the Monotone Convergence Theorem need not hold for decreasing sequences of functions.

8. Prove that the Lebesgue Outer measure is translation invariant, that is, for any set A and number :

$$y, m^*(A + y) = m^*(A)$$

****