A-0626

Total Pages : 4 **Roll No. ....**

MT–501

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Advanced Algebra–I)

1st Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Every homomorphic image of a group G is isomorphic to some quotient group of G. ?

2. A vector space V to be the direct sum of two of its subspaces w_1 and w_2 if and only if :

   (i) $V = w_1 + w_2$

   (ii) $w_1 \cap w_2 = \{0\}$

3. Define Kernel of homomorphism. Prove that Kernel of Homomorphism of module is a submodule.

4. Find the dual basis of the basis set :

$$B = \{(1, -1, 3), (0, 1, -1), (0, 3, -2)\}$$

   for $V_3(R)$.

5. If $\varepsilon\, \beta\, ,\alpha K$ are algebraic over F then $\beta\alpha\, ,\beta \pm \alpha$ and $\frac{\alpha}{\beta}(\beta \neq 0)$ are all algebraic over F.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Prove that two non zero elements of an integral domain are associates if :

$$\frac{a}{b} \text{ and } \frac{b}{a}$$

2. For what value of m, the vectors $(m, 3, 1)$ a linear combination of :

$$e_1 = (3, 2, 1) \text{ and } e_2 = (2, 1, 0)$$

3. Any product of Solvable group is solvable ?

4. If R be a ring such that $a^2 = a \ \forall \ a \in R$, Prove that :

   (i) $a + a = 0 \ \forall \ a \in R$

   (ii) $a + b = 0 \Rightarrow a = b$

5. If $G = \{a\}$ be a cyclic group of order 8. Find the quotient groups corresponding to the subgroups generated by a^2 and a^4.

6. Define the following :

   (i) Linear sum of submodules

   (ii) Quotient modules

7. Define the following :

   (i) Subnormal series

   (ii) Normal series

8. Prove that any nilpotent group is solvable.

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