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MAT-603

M.Sc. MATH (MSCMT)

(Operations Research)

3rd Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Prove that the necessary and sufficient condition for the existence and non-degeneracy of all the basic solutions of Ax = b is that every set of m columns of the augmented matrix [A, b] is linearly independent.

2. Using simplex method solve the following LPP :

$$Z = x_2 - 3x_3 + 2x_5$$

Maximize :

$$3x_2 - x_3 + 2x_5 \leq 7$$

Subject to :

$$-2x_2 + 4x_3 \leq 12$$

$$-4x_2 + 3x_3 + 8x_5 \leq 10$$

$$x_2, x_3, x_5 \geq 0$$

3. Solve the following mixed integer programming problem :

$$Z = 4x_1 + 6x_2 + 2x_3$$

Maximize :

$$4x_1 - 4x_2 \leq 5$$

Subject to :

$$-x_1 + 6x_2 \leq 5$$

$$-x_1 + x_2 + x_3 \leq 5$$

$$x_1, x_2, x_3 \geq 0$$

such that $x_1, x_3 \in Z$.

4. There are five persons available to perform five distinct tasks. The following table shows the amount of time (in hours) that each man needs to complete each task based on historical records :

		Jobs				
		I	II	III	IV	V
Men	A	2	9	2	7	1
	B	6	8	7	6	1
	C	4	6	5	3	1
	D	4	2	7	3	1
	E	5	3	9	5	1

Find the assignment of men to jobs that will minimize the total time taken.

5. Prove with the help of dynamic programming :

$$z = p_1 \log p_1 + p_2 \log p_2 + ... + p_n \log p_n$$

Subject to the constraints :

$$p_1 + p_2 + ... + p_n = 1$$

and $p_j \geq 0$ $(j = 1, 2, ..., n)$

Is a minimum when $p_1 = p_2 = ... = p_n = 1/n$.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Solve the linear programming using graphical method :

   Maximize : $Z = 30x_1 + 20x_2$

   Subject to :

   $$5x_1 + x_2 \leq 60$$

   $$x_1, x_2 \geq 0$$

   $$3x_1 + 4x_2 \leq 60$$

   $$4x_1 + 3x_2 \leq 60$$

2. Define the followings :

   (i) Slack variables

   (ii) Surplus variable

   (iii) Basic feasible solution

   (iv) Optimal solution

3. Using method of penalty (or Big M) solve the following LP problem :

Maximize :

$$Z = 2x_1 + 3x_2$$

Subject to :

$$x_1 + x_2 \geq 15$$

$$x_1 + 2x_2 \geq 6$$

$$x_1 + x_2 \geq 3$$

$$x_1, x_2 \geq 0$$

4. Find the dual of the following linear programming problem :

Maximize :

$$Z = 4x_1 + 6x_2 + 18x_3$$

Subject to :

$$x_1 + 3x_2 \geq 3$$

$$x_2 + 2x_3 \geq 5$$

$$x_1 \geq 0, x_2, x_3 \geq 0$$

5. In the given LPP :

Maximize :

$$Z = -x_1 + 2x_2 - x_3$$

Subject to :

$$3x_1 + x_2 - x_3 \leq 10$$

$$-x_1 + 4x_2 + x_3 \geq 6$$

$$x_2 + x_3 \geq 4$$

$$x_1, x_2, x_3 \geq 0$$

Establish the ranges for discrete modifications to the requirement vector's components b_2 and b_3 in order to preserve the feasibility of the present optimal solution.

6. Describe balanced transportation problem ? Discuss about its applications ?

7. Derive the necessary conditions for solving the non-linear programming problem :

Minimize :

$$Z = kx^{-1} y^{-2}$$

Subject to the constraints :

$$x^2 + y^2 - a^2 = 0 \text{ with } x \geq 0, y \geq 0$$

Hence find the value of Z.

8. Using the graphical method solve the non-linear programming problem :

Maximize

$$Z = x_1 + 2x_2$$

Subject to the constraints :

$$x_1^2 + x_2^2 \leq 1$$

$$2x_1 + x_2 \leq 2$$

and $x_1 \geq 0, x_2 \geq 0$

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