A-0614

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MAT-602

M.Sc. MATH (MSCMT)

(Functional Analysis)

3rd Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Prove that norm is a continuous function in a normed linear space.

2. Prove that every finite dimensional subspace Y of a normed space X is complete. In particular, every finite dimensional normed space is complete.

3. Consider a metric space X = (X, *d*), where X $\neq \phi$. Suppose that X is a complete and let T : X $\to$ X be a contraction on X. Then prove that T has precisely one (unique) fixed point.

4. Let X be an inner product space, let $x, y \in$ X then,

$$< x, y > = \frac{1}{4}[\| x+y \|^2 - \| x-y \|^2 + i \| x+iy \|^2 - \| x-iy \|^2]$$

5. Let (X, $\| \cdot \|$) be a normed linear space over a filed K (=R or C). Let X be a non-zero vector in *x*. then there is a bounded linear functional F on X such that :

   (a) F(x) = $\| x \|$

   (b) $\| F \| = 1$.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Let C^n is a vector space over C. for $x = (x_i)_{i=1}^{n} \in C^n$, define :

$$\| x \|_\infty = \max \{| x_1 |, | x_2 |, ..., | x_n |\} = \max_{1 \le i \le n} | x_i |$$

show that $\| \ C \|_p$ is a norm on C^n.

2. Prove that on a finite – dimensional vector space X, any norm $\|.\|$ is equivalent to any other norm $\| \cdot \|_0$.

3. Show that if a normed space X is finite dimensional, then every linear operator on X is bounded.

4. Show that the mapping T : $V_3(R) \to V_2(R)$ defined as :

$$T(a_1, a_2, a_3) = (3a_1 - 2a_2 + a_3, a_1 - 3a_2 - 2a_3)$$

is a linear transformation from $V_3(R)$ to $V_2(R)$.

5. Explain the statement of open mapping theorem and closed graph theorem.

6. Let X and Y be inner product spaces and $S : X \to Y$ be a bounded linear operator. Then prove that :

   (a) $S = 0$ if and only if $\langle Sx, y \rangle = 0$ for all $x \in X$ and $y \in Y$.

   (b) If $S : X \to X$, where X is over complex field, then $\langle Sx, x \rangle = 0$ for all $x \in X$ if and only if $S = 0$.

7. Define an isomorphism T of an inner product space X.

8. Let X = R set of reals). Define $\| x \| = | x |, \forall x \in R$. Prove that $(R, \| x \|)$ is a normed linear space.

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