A-0612

Total Pages : 4 **Roll No. .....**

MAT-509

MATHEMATICS (MSCMAT/MAMT)

(Mathematical Methods)

2nd Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Solve the following simultaneous differential equations by Laplace transform :

$$\frac{dx}{dt} + 4\frac{dy}{dt} - y = 0 \; ; \; \frac{dx}{dt} + 2y = e^{-t}$$

With the conditions $x(0) = y(0) = 0$.

2. Solve the Homogeneous Fredholm integral equation of the second kind :

$$y(x) = \lambda \int_0^{2\pi} \sin(x + t)\, y(t)\, dt$$

3. Find the resolvent kernel of the Volterra integral equation for the following kernel $k(x, t) = e^{x-t}$.

4. Find the Weirstrass function and test the extremal of the functional :

$$\mathrm{I}[y(x)] = \int_0^a y'^2 \, dx \quad \text{and } y(0) = 0,\ y(a) = b$$

where a > 0, b > 0.

5. Find the Fourier series to represent the function $f(x)$ given by :

$$f(x) = \begin{cases} x \ , & -\pi < x < 0 \\ -x \ , & 0 < x < \pi \end{cases}$$

and hence show that :

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...$$

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Find the Resolvent kernels for the Volterra integral equation :

$$y(x) = 1 + \int_0^x y(t)\, dt$$

and also find the solution.

2. Convert the initial value problem $y'' + y = f(x)$, $0 < x < 1$, $y(0) = y'(0) = 0$ into an integral equation.

3. Find the Laplace transform of

$$7e^{2t} + 9e^{-2t} + 5\cos t + 7t^3 + 5\sin 3t + 2$$

4. Find the inverse Laplace transform of :

$$\frac{6}{2p-3} - \frac{3+4p}{9p^2-16} + \frac{8-6p}{16p^2+9}$$

5. Show that the Homogeneous integral equation :

$$y(x) - \lambda\int_0^1 (3x-2)\, ty(t)\, dt = 0$$

has no characteristic number and eigenfunction.

6. Find the shortest distance between the point (1, 0) and the ellipse :

$$4x^2 + 9y^2 = 36$$

7. Solve the boundary value problem :

$$(1 - x^2)y'' - 2xy' + 2y = 0$$

subject to boundary conditions $y(0) = 0$, $y(1) = 1$ by Rayleigh-Ritz method.

8. Find the inverse Laplace transform of :

$$\frac{1+2p}{(p+2)^2(p-1)^2}$$

****