A-0610

Total Pages : 4 **Roll No. ....**

MAT-507

M.Sc. MATH (MSCMT)

(Measure Theory)

2nd Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) $2 \times 19 = 38$

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Prove that if $\{E_n\}$ be a countable collection of sets of real numbers. Then :

$$m^* (\cup_n E_n) \leq \Sigma_n m^* (E_n)$$

2. Prove that if f and g are two functions defined on the common domain E and f is measurable on E. If $f = g$ a.e. (almost everywhere) on E, then g is also a measurable function on E.

3. If f is a bounded function defined on a measurable set E, and $m(E) = 0$. Then show that :

$$\int_E f(x)\, dx = 0$$

4. Let X be an infinite set and B be the collection of all subsets A of X such that either A or A^c is finite. Show that B is an algebra but is not a σ-algebra.

5. Prove that if $\langle f_n \rangle$ is a sequence of nonnegative extended real valued measurable functions. Then $\int \underline{\lim} f_n \leq \underline{\lim} \int f_n$.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Define the following :

   (i) Hilbert space and example.

   (ii) Orthogonal elements.

   (iii) Orthonormal system.

   (iv) State Riesz-Fisher theorem.

2. Give yes/no that the following set are countable :

   (i) The set Q of a rational number.

   (ii) The set [0, 1].

   (iii) The set of all irrational number.

   (iv) The sets of natural numbers N and integers Z.

3. Prove that every bounded measurable function f defined on a measurable set E is L–integrable over E.

4. Proof that every open interval is a Borel set.

5. Proof that the intersection of two measurable sets is a measurable set.

6. Prove that the characteristic function ϕ_A of a set A is measurable if and only if A is a measurable set.

7. If a constant function on a measurable set E, where $f(x) = c$ for all $x \in$ E. Then prove that :

$$\int_{\mathrm{E}} f(x)\, dx = c.m\,(E)$$

8. Show that, for any set A and any $\in > 0$, there is an open set 0 containing A and such that :

$$m^* (0) \leq m^* (\mathrm{A}) + \in$$

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