A-0609

Total Pages : 4 **Roll No. ....**

MAT-506

M.Sc. MATH (MSCMT)

(Topology)

2nd Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Let $\{\mathbf{T}_\lambda : \lambda \in X\}$ where X is an arbitrary set, be a collection of topologies for X. Then prove that the intersection $\cap\{\mathbf{T}_\lambda : \lambda \in X\}$ is also a topology for X.

2. Let X and Y be topological spaces. Let function $f: X \to Y$. Then prove that the following are equivalent :

   (i) f is continuous.

   (ii) For every subset A of X, one has $f(\bar{A}) \subset \overline{f(A)}$.

   (iii) For every closed set B in Y, the set $f^{-1}(B)$ is closed in X.

3. Prove that Continuous image of connected space is connected.

4. Let $f : X \to Y$ be a continuous map of the compact metric space (X, d_x) to be metric space (Y, d_y). Then prove that f is uniformly continuous.

5. Prove that Every second countable space is separable space (X, **T**).

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Let X = $\{a, b, c\}$. Here are some collections of subsets of X, check that are topology or not ? Give the reason :

   (i) $\mathbf{T}_1 = \{\{a\}, \{c\}, \{a, b\}, \{a, c\}\}$.

   (ii) $\mathbf{T}_2 = \{\varnothing, \{a\}, \{b\}, X\}$.

   (iii) $\mathbf{T}_3 = \{\varnothing, \{a, b\}, \{a, c\}, X\}$.

2. Prove that with $d(x, y) = |x - y|$, the absolute value of the difference $x - y$, for each $x, y \in R$,(R , d) is a metric space.

3. Show that the real line is not compact.

4. Let X = $\{a, b, c\}$. $\mathbf{T} = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ Check that (X, **T**) is a Hausdorff space or not.

5. Prove that every compact subspace of a Hausdorff space is closed.

6. Show that R with usual topology is first countable.

7. Give an example of a basis for the Euclidean topological space (R^n, **T**).

8. Every closed and bounded interval on the real line is compact.

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