A-0608

Total Pages : 4 **Roll No. ....**

MAT-505

M.Sc. MATH (MSCMT)

(Advanced Linear Algebra)

2nd Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Consider a vector space R^3 (R). Then which of the following is/are subspace of R^3 (R) :

   (i) $W_1 = \{(x, y, z) : ax + by + cz = 0\}; a, b, c \in R$

   (ii) $W_2 = \{(x, y, z) : x \geq 0\}$

   (iii) $W_3 = \{(x, y, z) : x + y + z = 1\}$

2. Prove that two finite dimensional vector spaces over the same field are isomorphic if and only if they are of the same dimension.

3. Find the range, rank, null-space and nullity of the linear transformation $T : V_2(R) \rightarrow V_3(R)$, defined by $T(a, b) = (a + b, a - b, b)$.

4. State and prove the Bessel's inequality in a vector space.

5. Using the Gram-Schmidt orthogonalisation process to the vectors $S_1 = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ obtain an orthonormal basis for $R^3(R)$ with the standard inner product.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 5 & 5 & 6 \\ 3 & 7 & 6 & 11 \\ 4 & 8 & 4 & 12 \end{bmatrix}$.

2. Prove that the function $T : V_3(R) \to V_2(R)$ defined by :

$$T(a, b, c,) = (a, b) \ \forall \ a, b, c \in R$$

is a linear transformation.

3. Prove that the relation of similarity is an equivalence relation in the set of all linear transformations on a vector space V(F).

4. If T be the linear operator on R^3 defined by

$$T(x, y, z) = (3x + z, -2x + y, -x + 2y + 4z)$$

What is the matrix of T in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3,\}$ where $\alpha_1 = (1, 0, 1)$, $\alpha_2 = (-1, 2, 1)$, $\alpha_3 = (2, 1, 1)$?

5. If W be a subspace of a finite dimensional vector space V(F), then show that :

$$\dim\left(\frac{V}{W}\right) = \dim V - \dim W$$

6. Prove that a linear operator E on V is a projection if and only if I-E is a projection.

7. Prove that the vectors x and y in a real inner product space (Euclidean space) V are orthogonal if and only if :

$$\| x + y \|^2 = \| x \|^2 + \| y \|^2$$

8. Let S be any set of vectors in an inner product space V. Then show that $S^{\perp}$ is a subspace of V.

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