A-0607

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MAT–504

MATHEMATICS (MSCMAT/MAMT)

(Advanced Differential Equation-I)

1st Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Solve :

$$p^2 + 2py \cot x = y^2.$$

2. Solve :

(a) $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - 3y = x^2 + x$

(b) $(x^3 D^3 + 3x^2D^2 + xD + 1)y = x \log x.$

3. Using the Picard's method of successive approximations, find the third approximation of the solution of the equation :

$$\frac{dy}{dx} = x + y^2$$

4. (a) Find the orthogonal Trajectories of the series logarithmic spirals $r = a^\theta$.

(b) Explain the effect of damping on oscillatory motion.

5. Prove the recurrence relation for Chebyshev polynomials :

(a) $U_{n+1}(x) - 2xU_n(x) + U_{n-1}(x) = 0$

(b) $(1 - x^2)U'_n(x) = -nxU_n(x) + nU_{n-1}(x)$

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Solve :

   (a) $x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$

   (b) $(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0$

2. Define the following :

   (a) Non-Linear Differential Equation

   (b) Ordinary Differential Equation

   (c) Integrating factor

   (d) Degree of Differential Equation.

3. Solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ by using Variation of parameters.

4. Discuss the importance of Lyapunov constant.

5. Using the method by Krylov and Bogoliubov, solve the differential equation :

$$\frac{d^2y}{dt^2} + y + 2 \in \frac{dy}{dt} = 0$$

6. Using Rodrigue's formula, prove that :

$$P'_{n+1} - P'_{n-1} = (2n + 1)P_n$$

7. Expand $x^3 + x^2 - 3x + 2 = 0$ in the series of Laguerre's polynomials.

8. Prove that :

$$\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x) \text{ or } xJ'_n = nJ_n - xJ_{n+1}$$

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