A-0606

MAT–503

MATHEMATICS (MSCMAT/MAMT)

(Advanced Statistics)

1st Semester Examination, Session December 2024

Time : 2:00 Hrs.   Max. Marks : 70

Note :– This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

Section–A

Long Answer Type Questions   2×19=38

Note :– Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. In 2002 there will be three candidates for the position of principal–Mr. Chatter ji Mr. Ayangar and Dr. Singh- whose chances of getting the appointment are in the proportion 4 : 2 : 3 respectively. The probability that Mr.Chatterji if selected would introduce co-education in the college is 0.3 The probabilities of Mr. Ayangar and Dr. Singh doing the same are respectively 0.5 and 0.8 :

    (i) What is probability that there will be co-education in the college in 2003 ?

    (ii) If there is coeducation in the college in 2003, what is the probability that Dr. Singh is the principal.

2. Prove that, if F is distribution function of one dimensional random variable X, then :

$$F(-\infty) = \lim_{x \to -\infty} = F(x) = 0 \text{ and } F(\infty) = \lim_{x \to \infty} = F(x) = 1$$

3. A sample of 100 dry battery cell tested to find the length of life produced the following results : $\bar{x}$ = 12 hours, σ = 3 hours. Assuming the data to be normally distributed, what percentage of battery cells are expected to have life (i) More than 15 hours (ii) less than 6 hours (iii) between 10 and 14 hours.

4. The rank of same 16 students in Mathematics and Physics are as follows. Two numbers within brackets denote the ranks of the students in Mathematics and Physics :

(1,1) (2,10) (3,3) (4,4) (5,5) (6,7) (7,2) (8,6) (9,8) (10,11) (11,15) (12,9) (13,14) (14,12) (15,16) (16,13).

Calculate the rank correlation coefficient for proficiencies of this group in Mathematics and Physics.

5. Obtain the equations of the lines of regression for the following data. Also obtain the estimate of X for Y = 70.

X	Y
65	67
66	68
67	65
68	68
69	72
70	69
72	71

Section–B

Short Answer Type Questions   4×8=32

Note :– Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. What are the drawbacks of random sampling ?

2. It is believed that the precision (as measured by variance) of an instrument is no more than 0.16. Write down the null and alternative hypothesis for testing this belief. Carry out the test at 1% level, given 11 measurements of the same subject on the instrument :

    2.5, 2.3, 2.4, 2.3, 2.5, 2.7, 2.5, 2.6, 2.7, 2.5

3. A machinist is making engine parts with axle diameters of 0.700 inch. A random sample, of 10 parts shows a mean diameter of 0.742 inch with a standard deviation of 0.040 inch. Compute the statistic you would use to test whether the work is meeting the specifications. Also state how you would proceed further.

4. Let $x_1, x_2, ... x_n$ be a random sample from $N(\mu, \sigma^2)$ population. Find sufficient estimators for μ and σ^2.

5. Prove that :

$$P(E_1 \cap E_2^c) = P(E_1) - P(E_1 \cap E_2)$$

6. A card is drawn from a pack of 52 cards. Find the probability of getting a king or a red card.

7. A random variable X has the following probability function :

Values of X, x	$p(x)$
0	0
1	k
2	$2k$
3	$2k$
4	$3k$
5	k^2
6	$2k^2$
7	$7k^2 + k^2$

(i) Find k,

(ii) Evaluate $P(X < 6)$, $P(X \geq 6)$, and $P(0 < X < 5)$,

(iii) If $P(X \leq a) > \dfrac{1}{2}$, find the minimum value of a, and

(iv) Determine the distribution function of X.

8. The joint probability distribution of two random variables X and Y is given by :

$$P(X = 0, Y = 1) = \frac{1}{3}, \ P(X = 1, Y = -1) = \frac{1}{3}, \ \text{and}$$

$$P(X = 1, Y = 1) = \frac{1}{3}.$$

(i) Find Marginal distribution of X and Y, and

(ii) Conditional probability distribution of X given Y = 1.

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