A-0605

Total Pages : 3 **Roll No. ....**

MAT–502

MATHEMATICS (MSCMAT/MAMT)

(Real Analysis)

1st Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Prove that a sequence of real numbers converges iff it is Cauchy.

2. Let be a continuous function defined on closed interval $[a, b]$ Then is also uniformly continuous on $[a, b]$.

3. State and Prove Rolle's Theorem.

4. Union of arbitrary collection of open set is open in metric space.

5. A convergent sequence in a metric space is a Cauchy sequence.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Prove that the set :

$$\{x \in R : 0 < x \leq 1\}$$

is uncountable.

2. Prove that :

$$\lim_{n \to \infty} \left(\frac{1}{n} \right) = 0$$

3. Prove that $\sum \frac{1}{n}$ does not converge.

4. Explain Riemann Sums with the help of graph.

5. Examine the Convergence of :

$$\int_0^{\pi} \frac{1}{\sin x} dx$$

6. Let $f_n(x) = x^n$, $x \in [0,1]$. Then show that sequence of function $\{f_n(x)\}$ is not uniformly convergent on [0, 1].

7. Prove that every countable set is measurable.

8. Prove that the Cantor set has measure zero.

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