A-0604

Total Pages : 4 **Roll No. ....**

MAT–501

MATHEMATICS (MSCMAT/MAMT)

(Advanced Abstract Algebra)

1st Semester Examination, Session December 2024

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions $2 \times 19 = 38$

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. If G be a group and H is the subgroup of G. Then prove that the following statements are equivalent :

   (i) The subgroup H is normal in G.

   (ii) For all $a \in G$, $aHa^{-1} \subseteq H$.

   (iii) For all $a \in G$, $aHa^{-1} = H$.

2. If $f : G \to G'$ be a homomorphism then prove the following :

   (i) If e is the identity of G, then $f(e)$ is the identity of G'

   (ii) For any element $a \in G$, $f(a^{-1}) = [f(a)]^{-1}$

   (iii) If H is subgroup of G then $f(H)$ is subgroup of G'

   (iv) If order of any element $a \in G$ is finite then the order of $f(a)$ is divisor of the order of $a \in G$.

3. State and prove the necessary and sufficient condition for a non-empty subset K of field F to be subfield.

4. Prove that each integral domain can be imbedded in a field.

5. Show that following polynomials are irreducible over the indicated ring.

(i) $f(x) = x^3 + 2x^2 + x + 1$ over Q.

(ii) $f(x) = x^3 + 3x^2 - 6x + 3$ over Z

(iii) $f(x) = x^4 + x^2 + 1$ over $\frac{Z}{2Z}$

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Prove that each subgroup of cyclic group is normal.

2. Prove that every group of order P^2 is abelian.

3. Prove that A_5 is simple.

4. If G_1 and G_2 are two finite cyclic group of order m, n respectively. Then $G_1 \times G_2$ is cyclic if and only if $GCD(m, n) = 1$.

5. Prove that each group having order 15 is cyclic.

6. Let G be a group then G is nilpotent if and only if $G^{[n+1]} = \{e\}$ for some integer $n \geq 0$.

7. Prove that intersection of two ideal of a ring is also an ideal of the ring.

8. Prove that set of integer is an integral domain.

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