Roll No. ------------------

**MAMT-06**

**Analysis and Advanced Calculus**

MA/M.Sc. Mathematics (MAMT/MSCMT)

2ndYear Examination2024 (Dec.)

**TIME: 2 Hours Max Marks: 70**

Note: This paper is of Seventy (70) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

**SECTION – A**

**Long-answer - type questions**

**Note: Section ‘A’ contains Five (05) long-answer-type questions of Nineteen (19) marks each. Learners are required to answer any two (02) questions only. (2×19=38)**

1. State and prove the Minkowski’s Inequality.
2. If T be a linear transformation from a normed linear space N into the normed space , the prove that the following statements are equivalent.

**(i)** T is continuous

**(ii)** T is continuous at the origin i.e., $x\_{n}\rightarrow 0⇒T\left(x\_{n}\right)\rightarrow 0$.

**(iii)** T is bounded i.e., real  such that  for all .

1. State and prove Hahn-Banach theorem.
2. If$\left\{e\_{1},e\_{2},…,e\_{n}\right\}$ be a finite orthonormal set in a Hilbert space H and x be any vector in H, then prove the following

**(i)**  and **(ii)** 

1. Let f be a function on the interval  of R into R such that f is m times differentiable in [a,b] and (m+1) times differentiable in interval (a,b).Then prove that , where 

**SECTION – B**

**Short – answer – type questions**

**Note: Section ‘B’ contains eight (08) short- answer type questions of Eight (08) marks each. *Learners are required to answer any Four (04) questions only.* (4×8=32)**

1. Prove that every normed linear space is a metric space.
2. Prove that every compact subset of a normed space is bounded but the converse is not true.
3. If B and B′ be Banach spaces and if T is a continuous linear transformation of B onto B′ , then prove that T is an open mapping.
4. If x and y are any two vectors in an inner product space X , then prove that 
5. If x and y are any two orthogonal vectors in a Hilbert space H, then prove that 
6. An operator T onH is self-adjoint, then  and conversely.
7. If T is normal operator on a Hilbert space H, then eigenspaces of T are pairwise orthogonal.
8. Let f be a regulated function on a compact interval  of R into a Banach space X over K and g be a continuous linear map of X into a Banach space Y over K . Then prove that gof is regulated and .