

K-452

Total Pages : 3

Roll No.

MT-609

INTEGRAL EQUATIONS

MA/MSc Mathematics (MAMT/MScMT)

4th Semester Examination, 2023 (Dec.)

Time : 2 Hours]

[Max. Marks : 35

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half ($9\frac{1}{2}$) marks each. Learners are required to answer any Two (02) questions only.

($2 \times 9\frac{1}{2} = 19$)

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[P.T.O.

1. Form an integral equation corresponding to the differential equation

$$\frac{d^2y}{dx^2} + y = \cos x, \text{ with the initial conditions } y(0) = 0, y'(0) = 1.$$

2. Find the eigen values and eigen functions of the homogeneous integral equation

$$g(x) = \lambda \int_0^1 x t g(t) dt \text{ and solve it.}$$

3. Solve $g(x) = f(x) + \lambda \int_0^1 x t g(t) dt$.

4. Solve the integral equation

$$g(x) = \sin x - \frac{\pi}{4} + \frac{1}{4} \int_0^{\pi} x t g(t) dt.$$

5. Solve the following Volterra integral equation of the second kind $g(x) = x^2 + \int_0^x \sin(x-t) g(t) dt$.

SECTION-B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Prove that the Resolvent Kernel of $\sin(x - 2t)$ is equal to itself.
 2. $g(x) = f(x) + \lambda \int_0^1 xtg(t)dt$
 3. Prove that the Kernels $K(x, t) = xt$ and $L(x, t) = x^2t^2$ are orthogonal on $[-1, 1]$.
 4. Find the Resolvent Kernel for Volterra integral equation with the Kernel $K(x, t) = 1$.
 5. Find the Resolvent Kernel of the integral equation with the Kernel $K(x, t) = 2x$. Here $\lambda = 1$.
 6. Find the iterated Kernels for the Kernel $K(x, t) = e^x \cos t$, with $a = 0, b = \pi$.
 7. Discuss the differences between Volterra and Fredholm integral equations.
 8. Write a short note on various applications of integral equations.
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