# K-449

Total Pages : 4 Roll No. ......................

# MT-606

## ANALYSIS AND ADVANCED CALCULUS-II

MA/MSC Mathematics (MAMT/MSCMT)

4th Semester Examination, 2023 (Dec.)

**Time : 2 Hours]** **[Max. Marks : 35**

**Note :** This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

## SECTION–A

### (Long Answer Type Questions)

**Note :** Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only.

(2×9½=19)

**1.** Prove that if T be an operator on a Hilbert space H, then $\exists$ a unique linear operator $T^*$ on H such that $(Tx, y) = (x, T^* y)$ $\forall x, y \in$ H obviously $T^*$ is the adjoint operator H.

**2.** State and proof Spectral Theorem.

**3.** Define the following :

(i) Invariance.

(ii) Spectrum of an operator.

(iii) Spectral Theorem.

(iv) Directional derivative.

**4.** Prove that If P is a projection on a Hilbert space H with range M and the null space N, then $M \perp N$ iff P is self adjoint, and in this case $N = M^{\perp}$.

**5.** If X be a Banach space over the field K of scalars and let $f: [a, b] \to X$, $g: [a, b] \to R$ be continuous and differentiable functions such that $\|Df(t)\| \leq Dg(t)$ at each point $t \in (a, b)$. Then prove that $\|f(b) - f(a)\| \leq g(b) - g(a)$.

## SECTION–B

## (Short Answer Type Questions)

**Note :** Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Define the following :

   (i) Operator and functional

   (ii) Difference between Banach space and Hilbert space with example.

2. Define the following :

   (i) Linear operator with example.

   (ii) Jacobian Matrix with example.

3. Proof that if $x$ is an eigenvector of T, then $x$ cannot correspond more than one eigenvalue of T.

4. Let if X be a Banach space over the field K of scalars and V be an open subset of X. Suppose $f : V \to R$ be a function. Let $u$ and $v$ be any two distinct points in V such that $[u, v] \subset V$ and $f$ is differentiable at all points of $[u, v]$. Then proof that $f(v) - f(u) = Df(u + t(v - u)).\ (v - u)$ where $t \in (0, 1)$.

5. Define the following:

   (i) Continuously differentiable maps ($C^1$ – maps).

   (ii) Lipschitz's Property.

6. State the following theorem:

   (i) Taylor's theorem.

   (ii) Taylor' formula with Lagrange's Reminder.

**7.** State and proof Inverse function theorem.

**8.** Prove that T on a finite – dimensional Hilbert space H is singular $\Leftrightarrow$ there exist a nonzero vector $x$ in H such that $Tx = 0$.