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MT-604

INTEGRAL TRANSFORMS

MA/MSC Mathematics (MAMT/MSCMT)

1st Semester Examination, 2023 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. (a) Prove that if $\bar{f}(p)$ is the Laplace Transform of $f(t)$, then $\bar{f}(p - a)$ is the Laplace transform of $e^{at} f(t)$.

(b) Prove that if L[$f(t)$; p] = $\bar{f}(p)$ and a function $g(t)$ id defined as

$$g(t) = \begin{cases} f(t-a); t > a \\ 0; \quad t < a \end{cases}. \text{ Then } L[g(t); p] = e^{-ap}\bar{f}(p).$$

2. Apply convolution theorem to prove that

$$B(m, n) = \int_0^1 u^{m-1}(1-u)^{n-1} du = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} \text{ where } B(m, n)$$

is called Beta function. Deduce that

$$\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta = \frac{1}{2}B(m,n) = \frac{\Gamma(m)\Gamma(n)}{2\Gamma(m+n)}$$

3. Solve the boundary value problem

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}, \ x > 0 \text{ and } t > 0 \text{ with the boundary conditions}$$

$u(x, 0) = 0$ $u_t(x, 0) = 0$; $x > 0$

$u(0, t) = f(t)$, and $\lim_{x \to \infty} u(x, t) = 0$; $t \geq 0$

4. Find the Hankel transform of the function

$$f(x) = \begin{cases} a^2 - x^2, & 0 < x < a \\ 0, & x > a \end{cases}$$ taking $x J_0(px)$ as the kernel.

5. If the flow of heat is linear so that the variation of θ (temperature) with z and y-axes may be neglected and if it is assumed that no heat is generated in the medium, then solve the differential equation

$\frac{\partial \theta}{\partial t} = k \frac{\partial^2 \theta}{\partial x^2}$ where $\theta = f(x)$ and $-\infty < x < \infty$ when $t = 0$, $f(x)$ being a given function of x.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Find the laplace transform of

(a) $\frac{1 - \cos t}{t^2}$.

(b) $\sin at - at \cos at + \frac{\sin t}{t}$.

2. Find the inverse Laplace transform of $\frac{p}{(p^2+a^2)^2}$.

3. Solve $\frac{d^4y}{dx^4} - y = 1$, subject to conditons; $y(0) = y'(0) = y''(0) = y'''(0) = 0$.

4. Find $f(t)$, if its Fourier sine transform is $\frac{p}{1+p^2}$.

5. Show that $\int_0^\infty \frac{t^2 dt}{(t^2+a^2)} = \frac{\pi}{(2a)^5}, (a>0)$.

6. Prove that $M(f(x); p) = F(p)$, then $M(f(x^a); p) = \frac{1}{a} F\left(\frac{p}{a}\right)$, $a > 0$.

7. Find the Hankel transform of $f(x) = \begin{cases} 1, & 0 < x < a, \quad v > 0 \\ 0, & x > a, \quad v = 0 \end{cases}$

8. Define :

(a) Kernel of the Mellin transform

(b) Inverse Mellin transform