K-444

Total Pages : 3 Roll No. ....

MT-601

ANALYSIS AND ADVANCED CALCULUS-I

MA/MSC Mathematics (MAMT/MSCMT)

3rd Semester Examination, 2023 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Let X and Y be normed spaces over a same field K and let T: X $\rightarrow$ Y be a surjective linear operator. Then show that T is a topological isomorphism if and only if there exist constants k_1, k_2 such that $k_1 \| x \|_X \leq \| Tx \|_Y \leq k_2 \| x \|_X$. Recall that a topological isomorphism is a bijective continuous map whose inverse is also continuous.

2. State and prove the Riesz representation theorem.

3. State and prove Riesz Lemma.

4. State and prove the Hahn-Banach theorem for normed spaces.

5. If M is a linear subspace of a Hilbert space H, show that

$$\text{M is closed if and only if } M = M^{\perp\perp}.$$

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Define inner product space, and Hilbert space. Is every normed space an inner product space? Is every Banach space a Hilbert space? Justify your answers.

2. Define a bounded linear operator between two normed spaces. Show that a bounded linear operator is continuous.

3. Let $x = (5 + i, 7) \in C^2$. Find $\| x \|_p$ when $p = 1, 2, \infty$.

4. State the closed graph theorem. Give an example of a closed linear operator.

5. State and prove parallelogram equality.

6. Let $M \subset \mathbb{R}^2$, define the orthogonal complement $M^{\perp}$ of M in $\mathbb{R}^2$. Find orthogonal complement $M^{\perp}$ if $M = \{(1,2)\}$.

7. Show that an orthonormal set is linearly independent.

8. Give an example of an unbounded linear operator.