K-442

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MT-509

Differential Geometry and Tensor-II

MA/MSC Mathematics (MAMT/MSCMT)

2nd Semester Examination, 2023 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Derive the Cannonical equation of geodesic on the surface $r = r(u, v)$.

2. State and Prove Fundamental existence theorem for the surfaces.

3. (a) If a vector has components $\dot{x}, \dot{y}\left(\dot{x} = \frac{dx}{dt}, \dot{y} = \frac{dy}{dt}\right)$ in a rectangular Cartesian coordinates then $\dot{r}$ and $\dot{\theta}$ are its components in polar coordinates.

   (b) A covariant tensor of first order has components xy, $2y - z^2$, xz in rectangular coordinates. Determine its covariant component in spherical polar coordinates.

4. Prove that

   (a) $[ij, m] = g_{lm} \begin{bmatrix} l \\ ij \end{bmatrix}$.

   (b) $\dfrac{\partial g^{mk}}{\partial x^l} = -g^{mi}\begin{Bmatrix} k \\ il \end{Bmatrix} - g^{ki}\begin{Bmatrix} m \\ il \end{Bmatrix}$.

5. Prove that the necessary and sufficient condition that a system of coordinate be geodesic with pole P_0 are that their second covariant derivatives with respect to the metric of the space, all vanishes at P_0.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Prove that a curve on sphere is a geodesic if and only if it is great circle.

2. Prove that the curve $u = c$ (constant) is a geodesic iff $GG_1 + FG_2 - 2GF_2 = 0$.

3. Show that for the right helicoid $\vec{r} = (u \cos v, u \sin v, cv)$, $l = 0, m = 0, n = -u; \lambda = 0, \mu = \dfrac{u}{(n^2 + c^2)}, v = 0$.

4. Define :

(a) Kronecker delta.

(b) Contravariant vector.

5. If a metric of a V_3 is given by $ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6(dx^1)(dx^2) + 4(dx^2)(dx^3)$. Find (i) g (ii) g^{ij}.

6. If A^i is a contravariant vector then prove that *div* $A^i = \dfrac{1}{\sqrt{g}} \dfrac{\partial}{\partial x^i}\left(A^i \sqrt{g}\right)$.

7. If two unit vectors A^i and B^i are defined along a curve C such that their intrinsic derivative along C are zero. Show that the angle between them is constant.

8. Define :

(a) Flat space.

(b) Einstein tensor.