K-441

Total Pages : 4 Roll No. ....

MT-508

Special Functions

MA/MSC Mathematics (MAMT/MSCMT)

2nd Semester Examination, 2023 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Solve the Legendre's equation

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0.$$

2. Prove that $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$.

3. Prove that $J_{-n}(x)=(-1)^n J_n(x)$.

4. Prove that

(i) $2xH_n(x) = 2_nH_{n-1}(x) + H_{n+1}(x)$.

(ii) $H_n'(x) = 2nH_{n-1}(x)$ $(n \geq 1)$

5. Show that $\frac{e^{\frac{xt}{1-t}}}{1-t}=\sum_{n=0}^{\infty} L_n(x)t^n$.

SECTION–B
(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Solve in series

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0.$$

2. Prove that

$$ {}_2F_1\left[\frac{a}{2}, \frac{a}{2}+\frac{1}{2}; \frac{1}{2}; z^2\right] = \frac{1}{2}\left[(1-z)^{-a} + (1-z)^{a}\right]. $$

3. Prove that $(2n + 1)xQ_n = (n + 1)Q_{n+1} + nQ_{n-1}$.

4. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}$, sin x.

5. Expand x^n in a series of Hermite polynomials.

6. Prove that $L_n^k(x) = \frac{e^x x^{-k}}{n!} \frac{d^n}{dx^n}(e^{-x} x^{n+k})$.

7. Define

(a) Regular singular point.

(b) Radius of convergence.

8. Prove that

$$ \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\alpha(n, m) = \sum_{m=0}^{\infty}\sum_{n=0}^{\left[\frac{m}{2}\right]}\alpha(n, m-2n) $$

where the symbol $\sum_{n=0}^{\left[\frac{m}{2}\right]}$ indicates that n runs from 0 to the greatest integer less than or equal to m/2.
