K-440

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MT-507

TOPOLOGY

MA/MSC Mathematics (MAMT/MSCMT)

2nd Semester Examination, 2023 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only.

(2×9½=19)

1. What is a topology on a set $X \neq \phi$. Let $X = \{1,2,3\}$? Define four different topologies on X. Also compare these topologies and determine which topology is finer among all.

2. Define a subspace of a topological space. Consider $\mathbb{R}$ with the usual topology $\mathcal{T}_u$, and $Y = [0,1]$. Describe the subspace topology on Y in terms of its basis.

3. Define continuous maps between two topological spaces. Define a homeomorphism. Is every bijective continuous function a homeomorphism? Justify your answer.

4. Define a topological invariant property. Check if connectedness is a topological invariant property. Is boundedness a topological invariant property? Justify.

5. What do you mean by a convergent sequence in a topological space? Discuss the convergence of $\{1,2,3,...\}$ in the following topological spaces.

   - $\mathbb{R}$ with the indiscrete topology.
   - $\mathbb{R}$ with the discrete topology.
   - $\mathbb{R}$ with the co-finite topology.
   - $\mathbb{R}$ with the usual topology.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Define compact spaces. Is a non-empty discrete space always compact?

2. Define a T_3 space. Check if every T_3 space is a Hausdorff space. Justify.

3. Show that a one-to-one function from a co-finite topological space to itself is continuous.

4. Show that the continuous image of a compact set is compact.

5. Let X be a topological space and B_1; B_2 be two bases for the topology on X. Is $B_1 \cap B_2$ a base? Justify.

6. Prove that the product space of two connected spaces is connected.

7. Define quotient space of a topological space. Show that a continuous mapping of a compact space $(X, \mathcal{T})$ onto a Hausdorff space $(Y, \mathcal{T}_1)$ is a quotient mapping.

8. Let A be a subset of a topological space (X, $\mathcal{T}$). Show that A closed in (X, $\mathcal{T}$) if A contains all its limit points.