# K-439

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# MT-506

## ADVANCED ALGEBRA-II

MA/MSC Mathematics (MAMT/MSCMT)

2nd Semester Examination, 2023 (Dec.)

**Time : 2 Hours]** **[Max. Marks : 35**

**Note :** This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

## SECTION–A

### (Long Answer Type Questions)

**Note :** Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only.

(2×9½=19)

1. (a) Let F be a field, then prove that every polynomial of positive degree in $F[x]$ has splitting field,

   (b) Prove that every field of Characteristic zero is perfect.

2. Let G be a finite group of automorphism of a field K. Let F be the fixed field of G. *i.e.* $F = \{x \in K | \varphi(x) = x.$ for all $\varphi \in G\}$. Then prove that K is a Galois extension of F with $G\{K|F) = G$.

3. Let $V = \mathbb{R}^3$ and $t: V \rightarrow V$ be a linear map defined by $t(x, y, z) = \{x + z, -2x + y, -x + 2y + z)$. What is the matrix of $t$ with respect to basis $\{(1,1,-1), (-1,0,1), (1,2,1)\}$.

4. Let $B = \{b_1 = (1,0), b_2 = (0,1)\}$ and $B' = \{b_1' = (1, 3), b_2' = (2,5)\}$ be any two bases of $\mathbb{R}^2$. Then

   (a) Determine the transition matrix P from the basis B to the basis B'.

   (b) Determine the transition matrix Q from the basis B' to the basis B.

5. (a) State and Prove Pythagorean theorem in inner product space

   (b) Let V be an inner product space. Prove that for any two vector $v, u \in V$,

$$\left|\|u + v\|\right|^2 - \left|\|u - v\|\right|^2 = 4 <u,v>$$

## SECTION–B

### (Short Answer Type Questions)

**Note :** Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

**1.** Prove that any algebraic extension of finite field F is separable extension.

**2.** Prove that the each group of order 5 is isomorphic to the group $z_5$.

**3.** Prove that a linear transformation $t$: V $\rightarrow$ V is invertible if and only if matrix $t$ relative to some bases B of V is invertible.

**4.** Prove that an $n \times n$ square matrix A over field F is invertible iff rank(A) = $n$.

**5.** Prove that Similar matrices have the same characteristic polynomial and hence the same eigen values.

**6.** Prove that u = $(a_1, a_2)$, $v = (b_1, b_2) \in$ R$^2$, then, $< u, v \geq$ $a_1b_1 - a_2b_1 - a_1b_2 + 4a_2b_2$ defines an inner product space.

**7.** (a) State Bessel's inequality.

(b) Define orthonormal set with the help of example.

**8.** Prove that the eigen value of a self-adjoint linear transformation are real.