K-438

Total Pages : 4 Roll No. ....

MT-505

MECHANICS-I

MA/MSC Mathematics (MAMT/MSCMT)

1st Semester Examination, 2023 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Derive the general equation of motion of a rigid body using D'Alembert's Principle.

2. Show that the centre of suspension and centre of oscillation are convertible.

3. A uniform solid cylinder is placed with its axis horizontal on a plane, whose incilination to the horizon is a, show that the least coefficient of friction between it and the plane, so that is may roll and not slide, is $\frac{1}{3}\tan\alpha$. If the cylinder be hollow, and of small thickness, the least value is $\frac{1}{2}\tan\alpha$.

4. Derive Euler's dynamical equations of motion.

5. A rectangular parallelopiped whose edges are a, 2 a, 3 a can turn freely about is centre and is set rotating about a line perpendicular to the mean axis and making an angle $\cos^{-1}\frac{5}{8}$ with the least axis. Prove that ultimately the body will rotate about mean axis.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Deduce the general equations of motion of a rigid body from D' Alembert's principle.

2. A uniform triangular lamina ABC can oscillate in its own plane about an axis perpendicular to the plane of the lamina through the point A. Prove that the length of the simple equivalent pendulum is

$$\frac{3(b^2+c^2)-a^2}{4\sqrt{\{2(b^2+c^2)-a^2\}}}.$$

3. Deduce the principle of energy from the Lagrange's equations.

4. Prove that the kinetic energy of rigid body, moving in any manner is at any instant equal to the Kinetic Energy of the whole mass, supposed to be collected at its centre of inertia and moving with it.

5. Define :

(a) Principle of conservation of linear momentum under finite forces.

(b) Integral of energy and angular momentum.

6. The principal moments of inertia of a body at the centre of mass are A, 3A, 6A. The body is so started that its angular velocities about the axis are $3n$, $2n$, n respectively. If in the subsequent motion under no forces w_1, w_2, w_3 denote the angular velocities about the principal axis at time t, prove that

$$w_1 = 3w_3 = \frac{9n}{\sqrt{5}} \text{ sec } hu \text{ and } w_2 = 3n \tanh u$$

where $3nt + \frac{1}{2} \log 5.$

7. A cylinder rolls down a smooth plane whose inclination to the horizon is α, unwrapping as it goes a fine string fixed to the highest point of the plane, find its acceleration and the tension of the string.

8. Find the expression of kinetic energy in terms of the Motion of the Centre of Inertia and motion relative to Centre of Inertia.