K-436

MT-503

Differential Equation and Calculus of Variation

MA/MSC Mathematics (MAMT/MSCMT)

Ist Semester Examination, 2023 (Dec.)

Time : 2 Hours]   **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only.

$(2 \times 9\frac{1}{2} = 19)$

1. Show that there are two values of the constant for which $\dfrac{k}{x}$ is an integral $x^2\,(y_1 + y_2) = 2$, and hence obtain the general solution.

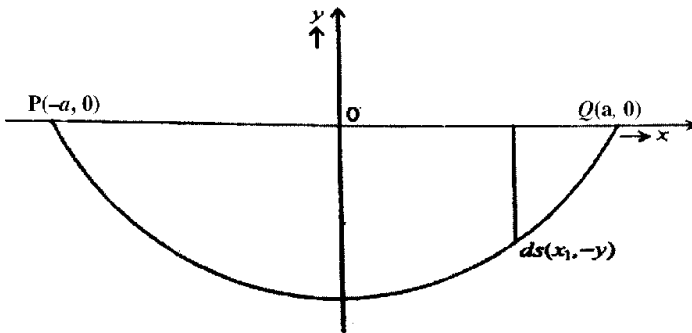2. $(x - y)(xr - xs - ys - yt) = (x + y)(p - q)$ by Monge's method.

3. Use the method of separation of variables to solve the PDE

$$4\dfrac{\partial u}{\partial t} + \dfrac{\partial u}{\partial x} = 3u, \text{ given that } u = 3e^{-x} - e^{-5x} \text{ when } t = 0.$$

4. Find the eigenvalues and eigenfunction for the boundary value problem

$$y'' - 2y + \lambda y = 0; \; y = (0), \; y(\pi) = 0.$$

5. Find the shape assumed by a uniform rope when suspended by its end from two points. At equal heights.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Solve $y(1 - \log y) \dfrac{d^2 y}{dx^2} + (1 + \log y) \left(\dfrac{dy}{dx} \right)^2 = 0.$

2. Solve $(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0.$

3. Classify the following differential equation as to type in the second quadrant of xy – plane.

4. Solve

    (a) $t + s + q = 0.$

    (b) $t - qx = x^2.$

5. Solve $\dfrac{d^2 y}{dx^2} = \sqrt{1 + \left(\dfrac{dy}{dx} \right)^2}.$

6. Show that the equation $u_{xx} + xu_{yy} + uy = 0$ is elliptic for $x > 0$ and hyperbolic for $x < 0$.

7. Define the following :

    (a)   Total differential equation.

    (b)   Laplace equation.

    (c)   Eigen value and Eigen function.

    (d)   Linear functional.

8. Find the characteristics of $y^2r - x^2t = 0$.

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