K-435

Total Pages : 3 Roll No. ....

MT-502

Real Analysis

MA/MSC Mathematics (MAMT/MSCMT)

1st Semester Examination, 2023 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only.

(2×9½=19)

1. For an algebra of sets \$, proof the following statement :

   (a) $\varnothing \in$ \$ and $X \in$ \$

   (b) \$ is closed under finite unions and intersections.

   (c) \$ is a semiring.

2. Proof that if E is a set such that $m^*(E) = 0$, then E is measurable.

3. If f is a bounded function defined on a measurable set E, and $m(E) = 0$. Then show that

$$\oint_E f(x)dx = 0.$$

4. State and proof Lebesgue monotone convergence theorem.

5. Define the term :

   (a) Orthogonal elements.

   (b) Orthonormal system.

   (c) Cantor set and it's properties.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Proof that every open interval is a Borel set.

2. Proof that the union of two measurable sets is a measurable set.

3. Show that a function f is measurable on a measurable set E if and only if its positive part f^+ and negative part f^- are measurable.

4. If a constant function on a measurable set E, where $f(x) = c$ for all $x \in$ E. Then prove that $\int_E f(x)dx = c.\, m(E)$.

5. State and proof Fatou's Lemma.

6. An orthonormal system $\{\varnothing_i\}$ is complete iff it is closed.

7. Show that space L_2 of square summable functions is a linear space.

8. State the following :

   (a) Almost everywhere property.

   (b) Lebesgue measure of a set.