K-434

Total Pages : 4 Roll No. ....

MT-501

Advanced Algebra-I

MA/MSC Mathematics (MAMT/MSCMT)

2nd Semester Examination, 2023 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only.

(2×9½=19)

1. (a) Let $G_1 \times G_2$ be two groups. Let $\cong G_1 \times G_2$.

$H_1 = \{\{a, e_2)|a \in G_1\} = G_1 \times \{e_2\}$ and $H_2 = \{(e_1, b)|b \in G_2\} = \{e_1\} \times G_2$ then prove that G is an internal direct product of H_1 and H_2

(b) If HK is the internal direct product of H and K, then

$$\frac{HK}{K} \cong H \text{ and } \frac{HK}{K} \cong K.$$

2. (a) Let (R, +) be the additive group of real number and (R+,.) be a multiplicative group of positive real numbers. Show that the mapping $\varphi : R \to R^+$ defined by $\varphi(x) = e^x$ is an isomorphism.

(b) Define

(i) Conjugate class.

(ii) Kernel of Homomorphism.

3. Prove that ring of Gaussian integer is Euclidean ring.

4. Let M and M^1 be two R-module. Then prove that the set $Hom_R(M, M^1)$ is an abelian group under pointwise addition of morphism.

5. Let V be a finite dimensional vector space over the field F, then there is a natural isomorphism of V onto V^{**}.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Let G be a finite group and $b \in$ G.

Then $o\,(C[a]) = \dfrac{o(G)}{o[N(a)]} = [G: N(a)]$

2. Show that S_n is non solvable for $n \geq 5$.

3. Let G be a group and H be a subgroup of G. Then H Δ G and G/H is abelian iff [G, G] subset of H.

4. Let a and b be two non zero element of Euclidean ring R such that b is not a unit in R, then $d(ab) = d(a)$.

5. Prove that ring R is an R-module over its subring R.

6. Show that the following mapping is linear t: $R^2 \to R^3$ given by $t(a, b) = (a + b, a - b, b)$, for all $(a, b) \in R^2$. Find range.

7. Define :

(a) Algebraic extension.

(b) Field extension.

8. State and Prove Sylvester's law of nullity.
