K-433

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MAT-504

ADVANCED DIFFERENTIAL EQUATION-I

Mathematics (MSCMAT/MAMT)

1st Semester Examination, 2023 (Dec.)

Time : 2 Hours] **Max. Marks : 70**

Note : This paper is of Seventy (70) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nineteen (19) marks each. Learners are required to answer any Two (02) questions only.

(2×19=38)

1. Solve $\frac{d^2y}{dx^2} + n^2y = secnx$ method by variation of parameters.

2. For the system of equations

$$\frac{dx}{dt} = 2x + y + xy^2, \frac{dy}{dt} = x - 2y - xy$$

Verify that (0, 0) is a critical point. Show that the system is almost linear and discuss the type and stability of the critical point (0, 0).

3. Show that the differential equation

$$\frac{d^2x}{dt^2} + \mu(x^2 - 1)\frac{dx}{dt} + = 0$$

has a periodic solution, μ is assumed to be a positive constant.

4. Discuss the role of Chebyshev Polynomials in solving partial differential equations.

5. If $L_n(x)$ to be the coefficient of t^n in the expansion of $\frac{1}{1-t} exp\left(\frac{xt}{1-t}\right)$, prove that

$$\int_0^{\pi/2} \frac{e^{-\tan\theta}}{\cos^2\theta} L_n(\tan\theta)\, L_m(\tan\theta) d\theta = \delta_{mn}.$$

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Eight (08) marks each. Learners are required to answer any Four (04) questions only. (4×8=32)

1. Find the differential equation satisfied by family of circles $x^2 + y^2 = a^2$, a being an arbitrary constant.

2. (a) Solve $(1 + x^2)dy = (1 + y^2)dx$.

(b) $\frac{dy}{dx} = xy + x + y + 1$.

3. Find the orthogonal trajectories of family of curves $y = ax^2$, a being parameter.

4. (a) Solve $(D^3 + D^2 + 4D + 4)y = 0$.

(b) Solve $\frac{d^2y}{dx^2} + (a + b)\frac{dy}{dx} + aby = 0$.

5. Discuss the stability of damped harmonic motion given by $\ddot{x} - 2\dot{x} + 2x = 0$.

6. Discuss the linear independence of two functions.

7. The quality factor Q of a tuning fork is 5×10^4. Find the value of time-interval after which its energy becomes 1/10 of its initial value.

8. Find the critical points of the System

$$\frac{dx}{dt} = y^2 - 5x + 6, \quad \frac{dy}{dt} = x - y.$$