

K-432

Total Pages : 5

Roll No.

MAT-503

ADVANCED STATISTICS

Mathematics (MSCMAT/MAMT)

1st Semester Examination, 2023 (Dec.)

Time : 2 Hours]

Max. Marks : 70

Note : This paper is of Seventy (70) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nineteen (19) marks each. Learners are required to answer any Two (02) questions only.

(2×19=38)

1. Suppose that a product is produced in three factories[^], Y and Z. It is known that factory X produces thrice as many items as factory Y, and that factories Y and Z produce the same number of items. Assume that it is known that 3 per cent of the items produced by each of the factories X and Z are defective while 5 per cent of those manufactured by factory Y are defective. All the items produced in three factories are stoked, and an item of product is selected at random.

- (a) What is the probability that this item is defective?
- (b) If an item selected at random is found to be defective, what is the probability that it was produced by factory X, Y and Z respectively?

2. A random variable X has the following probability function:

Values of X, x:	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k ²	2k ²	7k ² + k ²

- (i) Find k,
- (ii) Evaluate $P(X < 6)$, $P(X \geq 6)$, and $P(0 < X < 5)$,
- (iii) If $P(X \leq a) > \frac{1}{2}$, find the minimum value of a, and
- (iv) Determine the distribution function of X.

3. A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes.
4. The rank of same 16 students in Mathematics and Physics are as follows. Two numbers within brackets denote the ranks of the students in Mathematics and Physics :
- (1,1) (2,10) (3, 3) (4, 4) (5,5) (6,7) (7, 2) (8, 6) (9,8) (10,11)
(11,15) (12, 9) (13,14) (14,12) (15,16) (16,13).

Calculate the rank correlation coefficient for proficiencies of this group in Mathematics and Physics.

5. Prove that For any two events A and B, $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$.

SECTION-B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Eight (08) marks each. Learners are required to answer any Four (04) questions only. $(4 \times 8 = 32)$

1. A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn there is at least one ball of each other.

2. Let X be a discrete random variable having probability mass function

$$P_x x(x) = \begin{cases} 1/2 & \text{if } x = 1 \\ 1/3 & \text{if } x = 2 \\ 1/6 & \text{if } x = 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the third moment of X.

3. A sample of 100 dry battery cell tested to find the length of life produced the following results :

$$\bar{x} = 12 \text{ hours, } \sigma = 3 \text{ hours.}$$

Assuming the data to be normally distributed, what percentage of battery cells are expected to have life

- (i) More than 15 hours
 - (ii) Less than 6 hours
 - (iii) Between 10 and 14 hours.?
4. Two populations have their means equal, but S.D. of one is twice the other. Show that in the samples of size 2000 from each drawn under simple sampling conditions, the difference of means will, in all probability, not exceed 0.15σ , where σ is the smaller S.D. What is the probability that the difference will exceed half this amount?

5. It is believed that the precision (as measured by variance) of an instrument is no more than 0.16. Write down the null and alternative hypothesis for testing this belief. Carry out the test at 1% level, given 11 measurements of the same subject on the instrument :

2.5, 2.3, 2.4, 2.3, 2.5, 2.7, 2.5, 2.6, 2.7, 2.5.

6. A machinist is making engine parts with axle diameters of 0.700 inch. A random sample, of 10 parts shows a mean diameter of 0.742 inch with a standard deviation of 0.040 inch. Compute the statistic you would use to test whether the work is meeting the specifications. Also state how you would proceed further.

7. State and prove Baye's Theorem.

8. Prove that Two independent variables are uncorrelated.
