K-431

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MAT-502

ADVANCED REAL ANALYSIS

Mathematics (MSCMAT/MAMT)

lst Semester Examination, 2023 (Dec.)

Time : 2 Hours] **Max. Marks : 70**

Note : This paper is of Seventy (70) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nineteen (19) marks each. Learners are required to answer any Two (02) questions only.

(2×19=38)

1. Prove that every bounded sequence of real numbers has a convergent subsequence.

2. Let f be a continuous function defined on closed interval $[a, b]$. Then f is also uniformly continuous on $[a, b]$.

3. Prove that the necessary and sufficient condition for integrability of a bounded function f is for every $\varepsilon > 0$ there exists $\delta > 0$ such that for every partition P of $[a, b]$with norm $\mu(P) < \delta$ and $U(P, f) - L(P, f) < \varepsilon$.

4. Show that the sequence $\{f_n\}$, where $f_n(x) = \dfrac{n^2 x}{1 + n^3 x^2}$ is not uniformly convergent on $[0, 1]$.

5. Prove that union of arbitrary collection of open set is open in metric space.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Eight (08) marks each. Learners are required to answer any Four (04) questions only. (4×8=32)

1. Prove that the rational number system is not complete.

2. State and prove Rolle's Theorem.

3. Prove that if P_1 and P_2 are any two partitions of $[a, b]$ then $L(P_1, f, \alpha) \leq U(P_2, f, \alpha)$ and $L(P_2, f, \alpha) \leq U(P_1, f, \alpha)$.

4. Test the convergence of $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$.

5. Prove that if X is countable then $m^*(X) = 0$.

6. Prove that if A is a subset of the metric space (X, d), then $d(A) = d(\overline{A})$.

7. Show that Closure of connected set is connected.

8. Prove that if be a contraction on a complete metric space (X, d). Then f has a unique fixed point $u \in X$.
