K-430

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MAT-501

ADVANCED ABSTRACT ALGEBRA

Mathematics (MSCMAT/MAMT)

1st Semester Examination, 2023 (Dec.)

Time : 2 Hours] **Max. Marks : 70**

Note : This paper is of Seventy (70) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nineteen (19) marks each. Learners are required to answer any Two (02) questions only.

(2×19=38)

1. If G be a group and H is the subgroup of G. Then prove that the following statements are equivalent.

   (a) The subgroup H is normal in G.

   (b) For all $a \in$ G, $aHa^{-1} \subseteq$ H.

   (c) For all $a \in$ G, $aHa^{-1} =$ H.

2. Prove that, every group G is isomorphic to a permutation group. Also give a suitable example.

3. Prove that a group G having 3 conjugate classes either isomorphic to a cyclic group or isomorphic to S_3.

4. State and prove the Jordan-Holder theorem.

5. Any ring R without a unity element may be imbedded in a ring that contain a unity element.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Eight (08) marks each. Learners are required to answer any Four (04) questions only. (4×8=32)

1. Give an example of a group whose one of the subgroup make the normal subgroup while another subgroup does not form the normal subgroup (explain briefly).

2. If G be the finite group and Z(G) be the center of the group G. Then prove that the class equation of G can be written as,

$$O(G) = O[Z(G)] + \sum_{a \notin Z(G)} \frac{O(G)}{O[N(a)]}$$

where, summation runs over one element a in each conjugate class containing more than one element.

3. Prove that each group of order p^2 is abelian. What will you say about the group having order 121?

4. If G_1 and G_2 are two group of order m, n respectively. Then prove that $G_1 \times G_2$ is cyclic if and only if $gcd(m, n) = 1$.

5. Show that $< Q, + >$ has no maximal normal subgroup.

6. Prove that $(I_5, +_5, \times_5)$ is a field.

7. Every principal ideal domain is a unique factorization domain.

8. State and prove Eisenstein's criterion for irreducibility of polynomials over Q?