K-454

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MAMT-06

ANALYSIS AND ADVANCED CALCULUS

MA/M.Sc. Mathematics (MAMT/MSCMT)

2nd Year Examination, 2023 (Dec.)

Time : 2 Hours]   **Max. Marks : 70**

Note : This paper is of Seventy (70) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nineteen (19) marks each. Learners are required to answer any Two (02) questions only.

(2×19=38)

1. Prove that every normed linear space is a metric space.

2. Let N and N' be normed linear spaces over the same scalar field and let T be a linear transformation of N into N'. Then T is bounded if it is continuous.

3. State and prove Closed graph theorem.

4. State and prove Reisz lemma.

5. Prove that the space l_2^n consisting of all n tuples $x = (x_1, \ldots, x_n)$ of complex numbers and the inner product on l_2^n is defined as $<x, y> = \sum_{i=1}^{n} x_i \bar{y}_i$, where $y = (y_1, y_2, \ldots, y_n)$ is an inner product space.

SECTION–B
(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Eight (08) marks each. Learners are required to answer any Four (04) questions only.   (4×8=32)

1. Defined :
   (a) Banach space.
   (b) Hilbert space.

2. If x and y are any two vectors in a Hilbert space H, then prove that $\| x + y)^2\| + \|x - y\|^2 = 2(\| x \|^2 + \| y \|^2)$.

3. If M is a closed linear subspace of a Hilbert space H, then $H = M \oplus M^{\perp}$.

4. Define

5. (a) Adjoint operator.

    (b) Normal operator.

6. Prove that a closed linear subspace M of a Hilbert space H is invariant under an operater T iff $M^{\perp}$ is invariant under T^*.

7. Prove that every convergent sequence in a normed linear space is a Cauchy sequence.

8. Define regulated function and step function in a banach space.

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