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MAMT–09

M.A./M.Sc. Mathematics IInd Year Examination Dec., 2023

INTEGRAL TRANSFORMS AND INTEGRAL EQUATIONS

Time : 2 Hours] [**Max. Marks :** 70

Note* :–** This paper is of Seventy (70) marks divided into two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given there in. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Reduce the boundary value problem to Fredholm equation, $y'' + xy = 1$, subject to conditions :

$$y(0) = 0,\ y(1) = 0$$

2. Solve the integral equation nad discuss all its possible cases with the method of separable kernels :

$$u(x) = f(x) + \lambda \int_0^1 (1 - 3\xi) u(\xi) d\xi$$

3. State the prove convolution theorem for inverse Laplace transform.

4. Solve the homogeneous Fredholm integral equation of second kind :

$$y(x) = \lambda \int_0^{2\pi} \sin(x + t) y(t) dt$$

5. Use the Laplace transformation, solve the differentiable equation :

$$\frac{d^2y}{dx^2} + 9x = \cos, 2t$$

if $x(0) = 1,\ x\left(\frac{\pi}{2}\right) = -1.$

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Transform the initial value equation :

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

subject to conditions; $y(0) = 1$, $y'(0) = 0$ to Volterra integral equation.

2. Use the method of Laplace transformation to solve the integral equation :

$$u(x) = 1 + \int_0^x (\xi - x)u(\xi)d\xi$$

3. Solve the integral equation :

$$\sin x = \lambda \int_0^1 e^{x-\xi} u(\xi)d\xi$$

4. Find the eigen value and eigen function of :

$$y(x) = \lambda \int_0^1 e^x e^t y(t)dt$$

5. Using the convolution theorem, evaluate :

$$\int_0^t \sin u \cos(t-u)du$$

6. Using the method of successive approximation solve the integral equation, :

$$u(x) = x - \int_0^x (x - \xi) u(\xi) d\xi$$

7. Expand in a Fourier series the function $f(x) = x$ in the interval $0 < x \; 2\pi$.

8. Find the Hankel transform of the following :

(i) $\frac{\cos ax}{x}$

(ii) $\frac{\sin ax}{x}$

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