

MAT-501

Advanced Algebra

M. Sc. MATHEMATICS (MSCMAT-12)

First Year, Examination, 2017

Time : 3 Hours

Max. Marks : 80

Note : This paper is of **eighty (80)** marks containing **three (03)** Sections A, B and C. Learners are required to attempt the questions contained in these Sections according to the detailed instructions given therein.

Section-A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of nineteen (19) marks each. Learners are required to answer *two* (02) questions only.

1. Let H and N be two subgroups of G such that N is normal in G . Then $H \cap N$ is a normal subgroup of H and $\frac{H}{H \cap N} \cong \frac{HN}{N}$.
2. Let G be a group and $N < G$. If N and G/N are solvable then G/N is solvable.
3. Let V be a finite dimensional vector space over the field F then there is a natural isomorphism of V onto V^{**} .
4. State and prove Cayley-Hamilton theorem.

Section-B**(Short Answer Type Questions)**

Note : Section 'B' contains eight (08) short answer type questions of eight (08) marks each. Learners are required to answer *four* (04) questions only.

1. Let G_1 and G_2 be groups, then :

$$G_1 \times G_2 = G_2 \times G_1$$

2. Let R be a Euclidean ring. A non-zero $a \in R$ is a unit iff $d(a) = d(1)$, where 1 is the unity element of R .
3. Show that a left ideal M in a ring R is an R -module.
4. If $B = \{(1, 0), (0, 1)\}$ is the usual basis \mathbb{R}^2 . Determine its dual basis.
5. Let K be an extension of a field F . Then the elements of K which are algebraic over F form a subfield of K .
6. Any algebraic extension of a finite field F is a separable extension.
7. For any matrix A over a field F . $\text{Rank } A = \text{Rank } A^T$.
8. Any orthogonal set of non-zero vectors in an inner product space is linearly independent.

Section-C**(Objective Type Questions)**

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this Section are compulsory.

Write True/False in the following questions.

1. External direct product and internal direct product of same factors are isomorphic.
2. The centre of a group is abelian.

[3]

3. Every ring is a Euclidean Ring.
4. There are two binary operations defined in an R module.
5. A linear transformation maps zero to zero.

Fill in the blanks in the following questions.

6. Matrix A is orthogonal then $AA^T = \dots\dots\dots$.
7. Let A and B be any two similar matrices over the same field F, then $\det A = \dots\dots\dots$.
8. Being similar is an $\dots\dots\dots$ relation on the set of all $n \times n$ matrices having entries in the same field.
9. A field A is called $\dots\dots\dots$ field if all finite extensions of F are separable.
10. A linear transformation is also known as $\dots\dots\dots$.

